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CONFERENCE ON STOCHASTIC PROCESSES AND THEIR  
APPLICATIONS (16TH) HELD IN (U) STANFORD UNIV CA  
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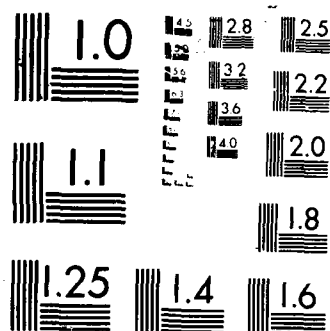
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16<sup>th</sup> Conference

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# ***Stochastic Processes and their Applications***



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***August 16-21, 1987  
Stanford University***



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**16th CONFERENCE ON STOCHASTIC PROCESSES  
AND THEIR APPLICATIONS**

**STANFORD, CALIFORNIA**

**AUGUST 17-21, 1987**

Organized under the auspices of the Bernoulli Society, with sponsorship and support from the following organizations:

Air Force Office of Scientific Research  
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Department of Operations Research, Stanford University  
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## Daily Schedule of Events

### STOCHASTIC PROCESSES AND THEIR APPLICATIONS

#### Sunday, August 16

12:00 - 6:00 p.m.	Room check-in (Sterling Quad) After 6:00 p.m. room check-in (Elliott Program Center)
2:00 - 9:00 p.m.	Conference Registration (Sterling Quad)
6:00 - 8:00 p.m.	Dinner (Sterling Quad)
7:30 - 9:30 p.m.	Informal Reception (grass area of Sterling Quad)

#### Monday, August 17

7:00 - 8:15 a.m.	Breakfast (Sterling Quad)
7:30 - 12:00 noon	Registration/ Information Desk (Terman Center)
8:45 - 10:00 a.m.	Invited Lectures (Terman Auditorium)
10:00 - 10:30 a.m.	Break (Terman Patio)
10:30 - 12:00 noon	Invited Lectures (Terman Auditorium)
12:15 - 1:15 p.m.	Lunch (Sterling Quad)
1:30 - 3:30 p.m.	Contributed Lectures
3:30 - 4:00 p.m.	Break (Garden)
4:00 - 5:35 p.m.	Contributed Lectures
6:00 - 8:00 p.m.	Formal Reception (Rodin Sculpture Garden)

#### Tuesday, August 18

7:00 - 8:00 a.m.	Breakfast (Sterling Quad)
8:00 - 12:00 noon	Registration/ Information Desk (Terman Center)
8:30 - 10:00 a.m.	Invited Lectures (Terman Auditorium)
10:00 - 10:30 a.m.	Break (Terman Patio)
10:30 - 12:00 noon	Invited Lectures (Terman Auditorium)
12:15 - 1:15 p.m.	Lunch (Sterling Quad)
1:30 - 3:30 p.m.	Contributed Lectures
3:30 - 4:00 p.m.	Break (Garden)
4:00 - 5:35 p.m.	Contributed Lectures
6:15 - 6:45 p.m.	Dinner Service (Sterling Quad)

### **Wednesday, August 19**

7:00 - 8:00 a.m. Breakfast (Sterling Quad)  
8:00 - 12:00 noon Registration/ Information Desk (Terman Patio)  
8:30 - 10:00 a.m. Invited Lectures (Terman Auditorium)  
10:00 - 10:30 a.m. Break (Terman Patio)  
10:30 - 12:00 noon Invited Lectures (Terman Auditorium)  
12:00 - 12:15 p.m. Pick up Box Lunches (Sterling Quad)  
12:30 Depart for Monterey (optional excursion)

### **Thursday, August 20**

7:00 - 8:00 a.m. Breakfast (Sterling Quad)  
8:00 - 12:00 noon Registration/ Information Desk (Terman Center)  
8:30 - 10:00 a.m. Invited Lectures (Terman Auditorium)  
10:00 - 10:30 a.m. Break (Terman Patio)  
10:30 - 12:00 noon Invited Lectures (Terman Auditorium)  
12:15 - 1:15 p.m. Lunch (Sterling Quad)  
1:30 - 3:30 p.m. Contributed Lectures  
3:30 - 4:00 p.m. Break (Garden)  
4:00 - 5:35 p.m. Contributed Lectures  
6:30 - 9:00 p.m. Conference Banquet (Inner Quad)

### **Friday, August 21**

7:00 - 8:00 a.m. Breakfast (Sterling Quad)  
8:00 - 12:00 noon Registration/ Information Desk (Terman Patio)  
8:30 - 10:00 a.m. Invited Lectures (Terman Auditorium)  
10:00 - 10:30 a.m. Break (Terman Patio)  
10:30 - 12:15 noon Invited Lectures (Terman Auditorium)  
12:30 - 1:15 p.m. Lunch (Sterling Quad)



## Invited Lectures Terman Auditorium

Monday, August 17			Session Chair
8:45- 9:00		Opening	N. Wessels
9:00-10:00	D. Kendall	New Developments in the Statistical Theory of Shape	D. Iglehart
10:00-10:30		Coffee Break	
10:30-11:15	P. Greenwood	Domains of Attraction for a Family of Processes Between Suprema and Sums	P. Jagers
11:15-12:00	T. Lindvall	Ergodicity and Inequalities for Certain Point Processes	
Tuesday, August 18			Session Chair
8:30- 9:15	P. Glynn	The Role of Poisson's Equation in Steady-State Simulation Output Analysis	D. Dawson
9:15-10:00	J. Walrand	Quick Simulations of Networks of GI/G/1 Queues	
10:00-10:30		Coffee Break	
10:30-11:15	D. Kennedy	Prophet Inequalities and Related Problems of Optimal Stopping	A. Joffe
11:15-12:00	M. Freidlin	Reaction-Diffusion Equations with Small Parameter: Probabilistic Approach	
Wednesday, August 19			Session Chair
8:30- 9:15	A. Mandelbaum	Stochastic Flow Networks	M. Harrison
9:15-10:00	T. Rolski	Queues with Non-Stationary Input Stream	
10:00-10:30		Coffee Break	
10:30-11:15	A. Barbour	Poisson Approximation by the Stein-Chen Method	D. Aldous
11:15-12:00	F. Kelly	Modeling Loss Networks	

Thursday, August 20			Session Chair
8:30- 9:15	S. Lalley	Traveling Waves in Branching Diffusion	R. Adler
9:15-10:00	T. Cox	Asymptotics for Finite Particle Systems	
10:00-10:30		Coffee Break	
10:30-11:15	C. Newman	Decomposition of Binary Random Fields, and Some Related Topics from Statistical Mechanics	C. Heyde
11:15-12:00	M. Steele	Probabilistic and Worst Case Analyses of Classical Problems of Combinatorial Optimization in Euclidean Space	
Friday, August 21			Session Chair
8:30- 9:15	B. Øksendal	When is a Stochastic Integral a Time Change of a Diffusion?	E. Çinlar
9:15-10:00	M. Yor	Principal Values of Brownian Local Times	
10:00-10:30		Coffee Break	
10:30-11:15	W. Vervaat	Algebraic Duality of Markov Processes	S. Karlin
11:15-12:15	K. L. Chung	Solving Boundary Value Problems by Probability Methods	

## OVERVIEW OF CONTRIBUTED PAPER SESSIONS

	Track A Room 380C Math Corner	Track B Room 370 Outer Quad	Track C Skilling Auditorium	Track D Room 320 Geology Corner
Session #1 Monday 1:30-3:30	Random Walks	Gaussian Processes and Related Topics	Estimation and Statistical Inference	Queueing Networks
Session #2 Monday 4:00-5:35	Markov Process Theory	General Theory of Stochastic Processes	Biological and Physical Science	Queueing and Related Models
Session #3 Tuesday 1:30-3:30	Diffusion Processes	Large Deviations and Extrema	Dynamic Optimization	Queueing Theory
Session #4 Tuesday 4:00-5:35	GSMP's and Insensitivity	Martingales and Related Theory	Economic Models	Queueing and Storage Models
Session #5 Thursday 1:30-3:30	Branching Processes and Population Models	Partial Sums and Renewal Processes	Simulation and Probabilistic Optimization	Reliability Theory and Related Topics
Session #6 Thursday 4:00-5:35	Particle Systems	Weak Convergence and Limit Theorems	Filtering Theory	Reliability and Detection Models

**Contributed Papers Track A**  
**Monday, August 17**  
**Room 380C, Math Corner**

**Session 1A**  
**Random Walks**  
**Chaired by J. Gani**

1:30-1:50	C. J. P. Belisle	Windings of Planar Random Walks
1:55-2:15	G. F. Lawler	Low Density Expansion for a Two-State Random Walk in Random Environment
2:20-2:40	W. den Hollander	A Limit Theorem for Random Walk in Random Scenery
2:45-3:05	C. Kluppelberg	Random Walks and Convolution Equivalent Distributions
3:10-3:30	L. A. K. Haneveld*	Random Walk on the Quadrant

**Session 2A**  
**Markov Process Theory**  
**Chaired by A. O. Pittinger**

4:00-4:20	F. Ball	Aggregated Markov Processes Incorporating Time Interval Omission
4:25-4:45	H. Kaspi	Random Time Changes for Processes with Random Birth and Death
4:50-5:10	J. B. Mitro	Homogeneous Random Measures for Markov Processes in Weak Duality: Study via an Entrance Boundary
5:15-5:35	G. I. Falin*	On a Random Walk with Internal Degrees of Freedom

\*Attendance uncertain at time of printing

**Contributed Papers Track B  
Monday, August 17  
Room 370, Outer Quad**

**Session 1B  
Gaussian Processes and Related Topics  
Chaired by L. A. Shepp**

1:30-1:50	S. M. Berman	Spectral Conditions for Local Nondeterminism
1:55-2:15	W. Bryc	Stochastic Processes with Linear Conditional Structure
2:20-2:40	A. T. Lawniczak	RKHS for Gaussian Measures on Metric Vector Spaces
2:45-3:05	M. S. Taqqu	Von Mises Statistics for Strongly Dependent Random Variables
3:10-3:30	R. Epstein*	Some Theorems of Central Limit Type for Markov Paths and Some Properties of Gaussian Random Fields

**Session 2B  
General Theory of Stochastic Processes  
Chaired by P. Protter**

4:00-4:20	D. P. Johnson	Space-Time Stochastic Processes
4:25-4:45	T. Norberg	An Existence Theorem for Measures on Partially Ordered Sets, with Applications to Random Set Theory
4:50-5:10	P. Ressel	Some Properties of Westcott's Functional
5:15-5:35	E. Willekens	Subordination of Stationary Processes

\*Attendance uncertain at time of printing

**Contributed Papers Track C**  
**Monday, August 17**  
**Skilling Auditorium**

**Session 1C**  
**Estimation and Statistical Inference**  
**Chaired by T. Sellke**

1:30-1:50	N. Becker	The Role of Martingales in the Analysis of Biomedical Data
1:55-2:15	C. C. Heyde	On Criteria of Optimality in Estimation for Stochastic Processes
2:20-2:40	P. A. Jacobs	On Estimating Measures of Performance for Queues and Survival Models
2:45-3:05	P. Spreij	Recursive Estimation for a Class of Counting Processes
3:10-3:30	B. R. Bhat*	Optimality of Sequential Probability Ratio Tests for a Class of Continuous Parameter Stochastic Processes

**Session 2C**  
**Biological and Physical Science**  
**Chaired by P. A. Jacobs**

4:00-4:20	L. A. Shepp	The Distribution of the Splitting Time for a DNA Strand
4:25-4:45	L. Bonilla	Nonequilibrium Phase Transitions and Systems of Infinitely Many Stochastic Equations
4:50-5:10	R. C. Blei*	$\alpha$ -Chaos
5:15-5:35	A. de Hoyos*	Statistical Alternatives for Measuring Functional Concordance Between Cerebral Hemispheres

\*Attendance uncertain at time of printing

**Contributed Papers Track D**  
**Monday, August 17**  
**Room 320, Geology Corner**

**Session 1D**  
**Queueing Networks**  
**Chaired by W. Whitt**

1:30-1:50	W. A. Massey	Lattice Bessel Functions and Their Applications to a Transient Analysis of Queueing Networks
1:55-2:15	M. I. Reiman	Light Traffic Derivatives via Likelihood Ratios
2:20-2:40	B. Simon	Anatomy of the Interpolation Method for Approximating Functions of Queueing Systems
2:45-3:05	J. M. Harrison	Brownian Models of Open Queueing Networks
3:10-3:30	R. J. Williams	Brownian Models of Open Queueing Networks: Product-Form Stationary Distributions

**Session 2D**  
**Queueing and Related Models**  
**Chaired by M. I. Reiman**

4:00-4:20	B. Pourbabai*	Tandem Behavior of a GI/G/1/0 Queueing Loss System
4:25-4:45	C. S. Tapiero*	Quality Control in Advanced Manufacturing Systems
4:50-5:10	D. Mitra*	The Transient Behavior in Erlang's Model for Large Trunk Groups and Various Traffic Conditions
5:15-5:35		No Paper Scheduled

\*Attendance uncertain at time of printing

**Contributed Papers Track A**  
**Tuesday, August 18**  
**Room 380C, Math Corner**

**Session 3A**  
**Diffusion Processes**  
**Chaired by R. J. Williams**

1:30-1:50	M. V. Day	Boundary Local Time and the Analysis of Small Parameter Exit Problems with Characteristic Boundaries
1:55-2:15	K. Saito	Stochastic Evolution Equations with Exponential Type Fluctuation
2:20-2:40	G. Kersting	On the Behavior of Solutions of Stochastic Differential Equations
2:45-3:05	D. McDonald	A Stochastic Differential Equation Involving Cylindrical Brownian Motion
3:10-3:30	W. S. Kendall*	Coupling and the Neumann Heat Kernel

**Session 4A**  
**GSMP's and Insensitivity**  
**Chaired by P. Glynn**

4:00-4:20	P. Taylor	Insensitivity Without Instantaneous Attention
4:25-4:45	W. Henderson	Insensitivity with Interruptions
4:50-5:10	M. Rumsewicz	Insensitivity and Generalised Transition Rates
5:15-5:35	M. Miyazawa	Remarks on the Basic Equations for a Supplemented GSMP and its Applications to Queues

\*Attendance uncertain at time of printing



**Contributed Papers Track B**  
**Tuesday, August 18**  
**Room 370, Outer Quad**

**Session 3B**  
**Large Deviations and Extrema**  
**Chaired by S. M. Berman**

1:30-1:50	D. Aldous	The Harmonic Mean Formula for Combinatorial and $D$ -Parameter Extrema
1:55-2:15	F. T. Bruss	On Invariant Record Processes and Their Applications
2:20-2:40	D. Pfeiffer	Strong Approximation of Records and Record Times by Poisson and Wiener Processes
2:45-3:05	G. Samorodnitsky	Maxima of Symmetric Stable Processes
3:15-3:30	A. Weiss	Large Deviations of Jump Markov Processes with Flat Boundaries

**Session 4B**  
**Martingales and Related Theory**  
**Chaired by M. S. Taqqu**

4:00-4:20	C.-S. Chou	On Some Inequalities of Semimartingales
4:25-4:45	M. Nikunen	On the Levy-Prohorov Distance Between Counting Processes
4:50-5:10	Y. Ohtsubo	Constrained Dynkin's Stopping Problem with Continuous Parameter
5:15-5:35		No Paper Scheduled

**Contributed Papers Track C**  
**Tuesday, August 18**  
**Skilling Auditorium**

**Session 3C**  
**Dynamic Optimization**  
**Chaired by A. F. Veinott, Jr.**

1:30-1:50	R. P. Kertz	Leaving an Interval in Limited Playing Time
1:55-2:15	G. Roberts	Some Stochastic Control Problems and Their Applications to Inequalities for Diffusions
2:20-2:40	R. J. Vanderbei	Optimal Switching Between a Pair of Brownian Motions
2:45-3:05	M. Fujisaki	Degenerate Bellman Equation and its Application
3:10-3:30	U. Yechiali	Stochastic Sequential Assignment Based on Discrete Match-Levels

**Session 4C**  
**Economic Models**  
**Chaired by D. Duffie**

4:00-4:20	I. Sahin	On a Class of Cumulative Processes in Warranty Analysis and Pension Accumulation
4:25-4:45	S. E. Shreve	Equilibrium in a Multi-Agent Consumption/Investment Problem
4:50-5:10	W. Willinger	A Pathwise Approach to Stochastic Integration
5:15-5:35	I. Karatzas*	Optimal Portfolio and Consumption Decisions for a "Small Investor" on a Finite Horizon

\*Attendance uncertain at time of printing

**Contributed Papers Track D  
Tuesday, August 18  
Room 320, Geology Corner**

**Session 3D  
Queueing Theory  
Chaired by J. de Smit**

1:30-1:50	W. Whitt	Central-Limit-Theorem Versions of $L = \lambda W$
1:55-2:15	E. Gelenbe	Recent Results on Queues with Time Variations
2:20-2:40	I. Mitrani	Stochastic Models of Queue Storage
2:45-3:05	K. Sigman	Queues as Harris Recurrent Markov Chains
3:10-3:30	A. Federgruen*	The Impact of the Composition of the Customer Base in General Queueing Models

**Session 4D  
Queueing and Storage Models  
Chaired by F. S. Hillier**

4:00-4:20	F. A. Attia	The Control of a Finit. Dam: Geometric Wiener Process Input
4:25-4:45	J. Gani	Problems in the Silting of Dams
4:50-5:10	S. Halfin	Response Times in M/M/1 Time Sharing Schemes with Limited Number of Service Positions
5:15-5:35	M. Kodama	Multichannel Queueing System with Semi-Ordered Entry

\*Attendance uncertain at time of printing

**Contributed Papers Track A**  
**Thursday, August 20**  
**Room 380C, Math Corner**

**Session 5A**  
**Branching Processes and Population Models**  
**Chaired by T. M. Liggett**

1:30-1:50	H. Cohn	On the Growth of the Multitype Supercritical Branching Process in a Random Environment
1:55-2:15	K. Yamada	Limit Theorems for Downcrossings of a Class of Birth and Death Processes
2:20-2:40	P. Jagers	Branching Processes as Markov Fields
2:45-3:05	L. Gorostiza	Limit Fluctuations of a Critical Branching Particle System in a Random Medium
3:10-3:30	N. K. Indira*	Normal Approximation in an Urn Model with Indistinguishable Balls

**Session 6A**  
**Particle Systems**  
**Chaired by H. Kaspı**

4:00-4:20	R. J. Adler	Fluctuation Theory for Systems of Signed and Unsigned Particles with Interaction Mechanisms Based on Intersection Local Times
4:25-4:45	T. M. Liggett	Systems of Independent Markov Chains
4:50-5:10	J. Horowitz	Random Measures and Particle Statistics
5:15-5:35		No Paper Scheduled

\*Attendance uncertain at time of printing

**Contributed Papers Track B**  
**Thursday, August 20**  
**Room 370, Outer Quad**

**Session 5B**  
**Partial Sums and Renewal Processes**  
**Chaired by M. Klass**

1:30-1:50	M. T. Alpuim	A Markovian Sequence: Stationarity and Extremal Properties
1:55-2:15	M. Csörgö	An Approximation of Stopped Sums with Applications in Queueing Theory
2:20-2:40	W.-J. Huang	On Some Characterizations of the Poisson Processes
2:45-3:05	F. W. Steutel	Infinitely Divisible Sequences and Renewal Sequences
3:10-3:30	H. Thorisson	The Initial Transience Problem: Solution in the Bounded Cycle Length Case

**Session 6B**  
**Weak and Convergence and Limit Theorems**  
**Chaired by H. Thorisson**

4:00-4:20	M. Harel	Weak Convergence of the Weighted Empirical Process Indexed by Rectangles Under Mixing Condition
4:25-4:45	M. Peligrad	On the Ibragimov-Iosifescu Conjecture for $\phi$ -Mixing Sequences
4:50-5:10	J. E. Yukich	Universal Limit Theorems for the Function Indexed Empirical Process
5:15-5:35	R. Pinsky	Some Limit Theorems for Diffusion Processes on the Circle and the Annulus

**Contributed Papers Track C**  
**Thursday, August 19**  
**Skilling Auditorium**

**Session 5C**  
**Simulation and Probabilistic Optimization**  
**Chaired by M. Steele**

1:30-1:50	V. Anantharam	Fast Simulation Techniques Based on Large Deviations Theory
1:55-2:15	Y. Chow	On the Convergence Rate of Annealing Processes
2:20-2:40	K. Schürger	A Multiparameter Almost Superadditive Limit Theorem and its Application to Combinatorial Optimization
2:45-3:05		No Paper Scheduled
3:10-3:30		No Paper Scheduled

**Session 6C**  
**Filtering Theory**  
**Chaired by T. Kailath**

4:00-4:20	F. Konecny	A Computational Approach to Nonlinear Filtering Based on Gauss Integration
4:25-4:45	H. Korezlioglu	White Noise Approach and Approximations for Two-Parameter Filters
4:50-5:10	E. Mayer-Wolf	Asymptotic Normality in Nonlinear Filtering via a Bayesian Cramer-Rao Inequality
5:15-5:35	H. N. Teodorescu*	Conditional and Pattern-Oriented Robustness

\*Attendance uncertain at time of printing

**Contributed Papers Track D  
Thursday, August 19  
Room 320, Geology Corner**

**Session 5D  
Reliability Theory and Related Topics  
Chaired by G. J. Lieberman**

1:30-1:50	E. Arjas	A Note on the Conditioning of the Survival Probability on Random Information
1:55-2:15	L. A. Baxter	The Stochastic Performance of Continuum Structure Functions
2:20-2:40	R. Grübel	Local Behavior of Simple Stochastic Models
2:45-3:05	S. R. Pliska	Optimal Inspection and Control of a Semi-Markov Deterioration Process
3:10-3:30	T. Ramalingam	On the Dependence Structure of Hitting Times of Multivariate Processes

**Session 6D  
Reliability and Detection Models  
Chaired by E. Arjas**

4:00-4:20	M. Abdel-Hameed	Optimal Replacement Policies of Devices Subject to a Pure Jump Markov Wear Process with Repair
4:25-4:45	M. Rumsewicz	A Spot Welding Reliability Problem
4:50-5:10	L. C. Thomas	Inspection Policies for Deteriorating Units with Symptomatic Emissions
5:15-5:35		No Paper Scheduled

**OPTIMAL REPLACEMENT POLICIES OF DEVICES  
SUBJECT TO A PURE JUMP MARKOV WEAR  
PROCESS WITH REPAIR**

by

**Mohamed Abdel-Hameed**

*Kuwait University  
Kuwait*

**ABSTRACT**

A system is subject to wear, the wear is due to shocks. The wear process is assumed to be an increasing pure jump process. Between shocks, the wear decreases due to a repair mechanism. Upon failure, the system is replaced by a new and identical one. The system can be also replaced before failure at a certain cost rate. We determine the optimal replacement time that minimizes the long-run average cost per unit time.



**FLUCTUATION THEORY FOR SYSTEMS OF SIGNED AND  
UNSIGNED PARTICLES WITH INTERACTION MECHANISMS  
BASED ON INTERSECTION LOCAL TIMES**

by

**Robert J. Adler**

*Israel Institute of Technology  
Haifa Israel*

**ABSTRACT**

We consider two distinct models of particle systems. In the first we have an infinite collection of identical Markov processes starting at random throughout Euclidean space. In the second a random sign is associated with each process. An interaction mechanism is introduced in each case via intersection local times, and the fluctuation theory of the systems studied as the processes become dense in space. In the first case the fluctuation theory always turns out to be Gaussian, irregardless of the order of the intersections taken to introduce the interaction mechanism. In the second case, an interaction mechanism based on  $k$ -th order intersections leads to a fluctuation theory akin to a  $\Phi^k$  Euclidean quantum field theory. We consider the consequences of these results and relate them to different models previously studied.

# THE HARMONIC MEAN FORMULA FOR COMBINATORIAL AND $d$ -PARAMETER EXTREMA

by

David Aldous

*University of California  
Berkeley*

## ABSTRACT

For a finite family  $(A_i; i \in I)$  of events satisfying a finite analog of stationarity, the elementary *harmonic mean formula* is

$$P(\cup A_i) = P(A_{i_0}) |I| E(N^{-1} | A_{i_0})$$

where  $A_{i_0}$  is an arbitrary event and  $N = \sum 1_{A_i}$  is the total number of events which occur. This leads to improvements of Boole's inequality in hard problems for which standard inclusion-exclusion techniques give no information. One domain of application is to combinatorial settings where the family grows exponentially: we illustrate with results about sparse random graphs. A different application is to maxima  $M_T = \sup\{X_t : t \in [0, T]^d\}$  of stationary  $d$ -parameter random fields. In the isotropic Gaussian case, for instance, it is known that the asymptotic behavior of  $M_t$  involves a constant  $c$  arising from the Gaussian field  $Z_t$  describing the local behavior of  $X_t$  around a high level. The harmonic mean formula leads to a proof of asymptotic convergence which is perhaps technically easier than standard proofs, and to a formula for  $c$  which is more amenable to analytic bounding and simulation.

# A MARKOVIAN SEQUENCE; STATIONARITY AND EXTREMAL PROPERTIES

by

Maria Teresa Alpuim

*University of Lisbon  
Portugal*

## ABSTRACT

Let  $\{Y_n\}$  be a sequence of i.i.d. random variables with common d.f.  $F(x)$  and  $X_0$  a random variable independent of the  $Y_i$ 's with d.f.  $F_0(x)$ . Define the Markovian sequence  $X_i = k \max\{X_{i-1}, Y_i\}$ , if  $i \geq 1$ ,  $X_i = X_0$ , if  $i = 0$ ,  $k \in R$ ,  $0 < k < 1$ . We study the main properties of this sequence and give conditions to obtain stationarity. It is shown that for any d.f.  $H(x)$  with left end point greater than or equal to zero for which  $\log H(e^*)$  is concave it is possible to construct such a sequence, being stationary and with marginal distributions equal to  $H(x)$ .

For this type of sequences the distribution of the maximum term and time between consecutive exceedances of a fixed level are trivial and we also give the distribution of the minimum. We study the limit law of extreme and  $k$ th order statistic in the stationary case and show that, in some cases, these sequences provide an interesting example of a stationary sequence verifying Leadbetter's  $D(u_n)$  condition and, thus, whose maximum term has a nondegenerated limit law, but possessing extremal index lesser than one for which it is possible to specify completely the limit laws for the  $k$ th order statistic.

Similar results hold when we consider sequences defined by  $Z_i = \max\{Z_{i-1}, Y_i\} - k$ ,  $k > 0$ , or even  $U_i = k \min\{U_{i-1}, Y_i\}$ ,  $k > 1$ , and  $V_i = \min\{V_{i-1}, Y_i\} + k$ ,  $k > 0$ .

**Keywords:** Markovian sequence, stationary distribution, limit laws, extreme values, order statistics.

# FAST SIMULATION TECHNIQUES BASED ON LARGE DEVIATIONS THEORY

by

V. Anantharam

*Cornell University  
Ithaca, NY*

## ABSTRACT

Let  $W_k$  denote the waiting time of customer  $k, k \geq 0$ , in an initially empty GI/G/1 queue. Fix  $a > 0$ . We prove weak limit theorems describing the behaviour of  $\frac{W_k}{n}, 0 \leq k \leq n$ , given  $W_n > na$ . Let  $X$  have the distribution of the difference between the service and interarrival distributions. We consider queues for which Cramer type conditions hold for  $X$ , and queues for which  $X$  has regularly varying positive tail.

A natural transient performance criterion for a network design is the probability of the event that any one of the first  $N$  customers entering the initially empty network incurs a delay of at least  $T$  seconds.  $N$  and  $T$  are specified to the designer. We study the Monte Carlo simulation of this criterion for a tandem of GI/G/1 queues with renewal arrivals, under Cramer type conditions on the interarrival and service distributions. We describe a technique to speed up the simulation of this criterion, which is asymptotically optimal in a certain sense.

# A NOTE ON THE CONDITIONING OF THE SURVIVAL PROBABILITY ON RANDOM INFORMATION

by

**Elja Arjas**

*University of Oulu  
Finland*

and

**A. Yashin**

*Institute of Control Sciences  
Moscow, USSR*

## ABSTRACT

Failure intensities in which the evaluation of hazard is based on the observation of an auxiliary random process have become increasingly popular in survival modeling. While their definition in the counting process and martingale framework is well known, their relationship to conditional survival functions does not seem to be equally well understood. This paper gives a set of necessary and sufficient conditions for the so called exponential formula in this context.

# THE CONTROL OF A FINITE DAM: GEOMETRIC WIENER PROCESS INPUT

by

F. A. Attia

*Kuwait University  
Kuwait*

## ABSTRACT

The resolvent operator  $R_\alpha$  of the diffusion process  $Y(t) = \exp\{X(t)\}$ , where  $X(t)$  is a Wiener process with mean  $\mu$  and variance  $\sigma^2$ , is determined. The associated kernel  $K_\alpha(x, y)$  is also obtained. These results are then used to determine the long-run average cost of operating a finite dam with the cumulative input process  $Y(t)$ . The system is controlled by a  $P_{\lambda, r}^M$  policy (Attia, 1987).

# AGGREGATED MARKOV PROCESSES INCORPORATING TIME INTERVAL OMISSION

by

Frank Ball

*University of Nottingham  
England*

## ABSTRACT

We consider a finite state space continuous time Markov chain that is time reversible. The complete process is not observable but rather the state space is partitioned into two classes, termed "open" and "closed", and it is only possible to observe which class the process is in. Such aggregated Markov processes have found considerable application in the modelling and analysis of single channel records that occur in certain neurophysiological investigations. The open and closed states referred to above correspond to the receptor channel being open or closed. A further problem with single channel analysis is that short sojourns in either the open or closed states are unlikely to be detected, a phenomenon known as time interval omission.

We show that the dynamic stochastic properties of the observed process (incorporating time interval omission) are completely described by an embedded Markov renewal process, whose parameters are obtained. We derive expressions for the moments and autocorrelation functions of successive observed sojourns in both the open and closed states, and also for a measure of the temporal clustering of observed channel openings. Finally we illustrate our theory by describing and analysing a model for the gating mechanism of the locust (*Schistocerca gregaria*) muscle glutamate receptor.

# POISSON APPROXIMATION BY THE STEIN-CHEN METHOD

by

A. D. Barbour

## ABSTRACT

Stein (1970) introduced a new technique for obtaining rates of convergence to the normal distribution, and applied it to the central limit theorem for stationary mixing sequences. His method is, however, not restricted to normal convergence, but can be adapted for use in a variety of other contexts. In particular, Chen (1975) showed how to use it for Poisson approximation.

It transpires that the method is more naturally suited to Poisson than to normal approximation. This is partly because the quantities which have to be estimated, in order to obtain a rate of convergence, are simpler in the Poisson case. The main reason, however, is that the metric which arises from Stein's method for the Poisson distribution, the total variation metric, is widely used, whereas that for the normal distribution, based on expectations of smooth functions of random variable, is not popular, and more work is required to translate the results obtained into convergence rates in the more common metrics.

In this paper, the Stein-Chen method for Poisson approximation is outlined, and is illustrated with reference to a variety of examples. In the easiest case, that of independent 0-1 summands, very good results are obtained rather simply. However, the chief attraction of the method lies in its applicability to a variety of problems concerning sums of dependent random variables. Broadly speaking, the method proves effective where the dependence is in some sense local, as for stationary mixing sequences and dissociated arrays, or diffuse, as in combinatorial and exchangeable applications, and seems less suitable where a natural flow of time is present. In this sense, the approach is complementary to that through martingales.

The success of the method in applications depends on how the argument is carried through, and in particular on careful choice of an appropriate coupling. However, the fact that, in many cases, optimal convergence rates can be obtained, makes Stein's method a powerful adjunct to the other techniques available.



# THE STOCHASTIC PERFORMANCE OF CONTINUUM STRUCTURE FUNCTIONS

by

Laurence A. Baxter

State University of New York at Stony Brook  
Stony Brook, NY

## ABSTRACT

A *continuum structure function* (CSF) is a nondecreasing mapping  $\gamma : [0, 1]^n \rightarrow [0, 1]$ . Such functions are of interest in reliability theory where  $x_i$  denotes the state of component  $i$  ( $i = 1, 2, \dots, n$ ) and  $\gamma(\underline{x})$  denotes the state of the system composed of components  $\{1, 2, \dots, n\}$ : see Baxter (1984) *J. Appl. Prob.* 802-815, Baxter (1986) *Math. Proc. Camb. Phil. Soc.* 331-338 for details.

Define  $P_\alpha = \{\underline{x} | \gamma(\underline{x}) \geq \alpha \text{ whereas } \gamma(\underline{y}) < \alpha \text{ for all } \underline{y} < \underline{x}\}$ ,  $0 < \alpha \leq 1$ , the set of *minimal vectors* to level  $\alpha$ . Block and Savits (1984) *Operat. Res.* 703-714 show that any right-continuous CSF is characterized by its minimal vector sets.

Suppose, now, that  $X_1, \dots, X_n$ , the states of the components, are independent random variables. In general, the distribution of  $\gamma(\underline{X})$  is hard to calculate. However, if each  $P_\alpha$  is finite, the distribution is easily evaluated. Further, if  $\gamma$  is an arbitrary right-continuous CSF, the distribution of  $\gamma(\underline{X})$  can be approximated arbitrarily closely by that of  $\gamma'(\underline{X})$  where  $\gamma'$  is a CSF for which each  $P_\alpha$  is finite. If the distribution function of  $\gamma(\underline{X})$  is continuous, the convergence is uniform.

Suppose, now that the states of the components comprise a stochastic process  $\{\underline{X}(t), t \geq 0\}$  such that  $\underline{X}(t) \xrightarrow{D} \underline{X}$  as  $t \rightarrow \infty$ . Further, the system is assumed to change in time, the CSF at time  $t$  being denoted  $\gamma_t$ . If  $\gamma_t \rightarrow \gamma$  either (i) pointwise or (ii) in a quasi-Skorohod topology, then  $\gamma_t(\underline{X}(t)) \xrightarrow{D} \gamma(\underline{X})$  as  $t \rightarrow \infty$ .

# THE ROLE OF MARTINGALES IN THE ANALYSIS OF BIOMEDICAL DATA

by

Niels Becker

*La Trobe University  
Bundoora, Australia*

## ABSTRACT

The theory of martingales has become a useful tool in the construction of estimating equations, and associated inference problems. It is particularly useful for non-parametric inference, but is increasingly found to be useful in parametric inference for processes which are only partially observed. The present paper reviews applications in the analysis of infectious disease data, survival/sacrifice data and capture-recapture experiments.

# WINDINGS OF PLANAR RANDOM WALKS

by

Claude J. P. B  lisle\*

*The University of Michigan  
Ann Arbor, MI*

## ABSTRACT

Let  $X_1, X_2, X_3, \dots$  be a sequence of i.i.d.  $\mathbb{R}^2$ -valued bounded random variables with mean vector zero and covariance matrix identity. Let  $S = (S_n; n = 0, 1, 2, \dots)$  be the random walk defined by  $S_n = \sum_{i=1}^n X_i$ . Let  $\phi(n)$  be the winding of  $S$  at time  $n$ , i.e. the total angle wound by  $S$  around the origin up to time  $n$ . Under certain regularity and symmetry conditions on the distribution of  $X_1$ , we show that the distribution of  $2\phi(n)/\log n$  converges to the distribution with density  $1/(2 \cosh(\pi\omega/2))$ .

*Keywords and phrases:* Planar random walks, planar Brownian motion, windings, weak and strong invariance principle, asymptotic distributions.

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# SPECTRAL CONDITIONS FOR LOCAL NONDETERMINISM

by

Simeon M. Berman

*Courant Institute of Mathematical Sciences  
New York University, New York*

## ABSTRACT

Let  $X(t)$  be a real Gaussian process with stationary increments and spectral distribution function  $F(x)$ . Put  $\phi(t) = F(\infty) - F(1/t)$ ,  $t > 0$ . Sufficient conditions in terms of  $F$  are given for the process to be locally  $\phi$ -nondeterministic. These are formulated for discrete and absolutely continuous functions  $F$ . The results in the discrete case are applied to the analysis of the local time of a random Fourier series with i.i.d. coefficients. The class of distributions of the coefficients includes not only the normal distribution but others such as the symmetric stable distribution.

**OPTIMALITY OF SEQUENTIAL PROBABILITY  
RATIO TESTS FOR A CLASS OF CONTINUOUS  
PARAMETER STOCHASTIC PROCESSES**

by

**B. R. Bhat**

and

**Indira Devi A**

*Karnatak University  
Dharwad. India*

**ABSTRACT**

For a wide class of stochastic processes (s.p.) including processes belonging to exponential families, it is proved that Wald SPRT is optimal in the sense of minimizing the expectation of an increasing process associated with the s.p.

*Keywords:* Wald SPRT, continuous parameter s.p., optimal property.

# $\alpha$ -CHAOS

by

Ron C. Blei\*

*University of Connecticut  
Storrs, CT*

## ABSTRACT

Although presented from two different vantage points, Einstein's description of Brownian movements (c. 1905) and Wiener's "differential-space," a model for the same (c. 1923), are fundamentally related through the basic assumption that displacements of a Brownian particle are mutually independent phenomena. Based on 'independence,' both models were idealized descriptions. Our present work is an attempt to make precise, in a mathematical context, the notion of interdependencies between Brownian displacements.

The basic idea is that interdependencies within the  $L^2$ -span of an orthonormal system can be measured by an exponent  $\alpha \in [1, \infty)$  characterizing an asymptotic growth of  $L^p$ -norms of elements in that span: the parameter  $\alpha$  in effect registers an asymptotic stability of distributions in the  $L^2$ -span of the given system. After formalizing the notion of sub- $\alpha$ -systems, one defines  $\alpha$ -chaos to be processes whose increments are sub- $\gamma$ -systems if and only if  $\gamma > \alpha$ . (The classical Wiener process is a 1-chaos.) Every  $\alpha$ -chaos is a stochastic integrator with corresponding stochastic series representing sample paths that are almost surely continuous and of unbounded variation. The existence of  $\alpha$ -chaos is established essentially via the existence of  $\alpha$ -dimensional lattice sets. Indeed, a concrete version of  $\alpha$ -chaos in discrete time can be simulated for every  $\alpha$  within a framework of the finite Fourier transform.

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\* Research partially supported by NSF grant DMS-8601485.

# NONEQUILIBRIUM PHASE TRANSITIONS AND SYSTEMS OF INFINITELY MANY STOCHASTIC EQUATIONS

by

Luis L. Bonilla

## ABSTRACT

A nonlinear Fokker-Planck equation (FPE) is derived to describe the cooperative behavior of general stochastic systems interacting via mean-field couplings, in the limit of an infinite number of such systems. In the weak noise limit a general result yields the possibility of having bifurcations from stationary solutions of the nonlinear FPE into stable time-dependent solutions. The latter are interpreted as nonequilibrium probability distribution (states) and the bifurcations to them as nonequilibrium phase transitions in the thermodynamic limit, results for a model of self-synchronizing nonlinear oscillators are given as illustrations. For this model, a Hopf bifurcation to a time-periodic probability density is described for any value of the noise strength. A recent extension of this result to the stochastic nonlinear parabolic equation, which the mean-field model caricaturizes, is also given. In this case, a new (formal) treatment which includes the use of the renormalization group is necessary. Part of these results have appeared in

- L. L. Bonilla, *J. Statistical Physics* 46, 659-678 (1987).
- L. L. Bonilla, J. M. Casado & M. Morillo, Self-synchronization of populations of nonlinear oscillators in the thermodynamic limit, *J. Stat. Phys.* 48, (1/2), July 1987.

# ON INVARIANT RECORD PROCESSES AND THEIR APPLICATIONS

by

F. Thomas Bruss

*Facultés Universitaires Notre-Dame de la Paix,  
Namur, Belgium*

## ABSTRACT

Let  $A_1, A_2, \dots$  be independent random variables with common continuous distribution function  $F$  defined on some finite or infinite interval  $[0, t]$ , and  $N$  be some  $N$ -valued random variable independent of the  $A_i$ 's.

Let  $T_1 < T_2 < \dots < T_N$  be the order statistics of  $A_1, \dots, A_N$ . The  $A_i$ 's are thought of as being random arrival times of objects drawn independently from a (not necessarily known) distribution,  $N$  as the total number of arrivals on  $[0, t]$  and the  $T_i$ 's as the chronological order in which an observer can see them. We say that  $T_j$  is a  $k$ -record, if the associated object is the  $k$ th best of the first  $j$ . The problem is to predict, having observed  $[0, \tau]$ ,  $\tau < t$ , the total number of future  $k$ -records in any subinterval of  $[\tau_1, \tau_2]$  with  $\tau \leq \tau_1 < \tau_2 \leq t$ .

Suppose that the observer, not knowing  $N$  or its distribution, starts, at time 0, with a noninformative prior (e.g.  $P(N = n) = c, \forall n$ ). If we denote by  $\sigma_\tau$  the  $\sigma$ -field generated by arrivals (or records) up to time  $\tau$  then clearly  $P(N = n | \sigma_\tau)$  depends on  $\sigma_\tau$ . We will, however, prove the following somewhat surprising result.

**Theorem.** Let  $R_k^\tau(s) = \# k$ -records in  $(\tau, s]$ . Then, given  $T_j \leq \tau$ , all record counting processes  $(R_1^\tau(s)), (R_2^\tau(s)), \dots, (R_{j+1}^\tau(s))$  are i.i.d. inhomogeneous Poisson processes with intensity  $1/F(s)$  on  $[\tau, t]$ .

Applications show how several optimal selecting problems can be reduced to easy exercises simply by imbedding them into this model.

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# STOCHASTIC PROCESSES WITH LINEAR CONDITIONAL STRUCTURE

by

Włodzimierz Bryc

*University of Cincinnati  
Cincinnati, OH*

## ABSTRACT

Let  $T$  be a set interpreted as "time" (finite, countable or continuum).

Consider a class of integrable stochastic processes  $(X_t)_{t \in T}$  with the following property. For each  $t \in T$  and each finite set  $F \subset T$ , there are numbers  $b, \{a_j\}_{j \in F}$  (which depend on  $t, F$  and the finite dimensional distributions of the process  $(X_t)$ ) such that

$$(1) \quad E(X_t | X_s : s \in F) = b + \sum_{j \in F} a_j X_j.$$

For the purpose of this talk, we shall say that such processes have "linear conditional structure". (This definition is motivated by random fields rather than stochastic processes – we don't use any ordering of  $T$ . Parallel version with  $T$  ordered and  $F \subset T$  finite such that  $s \leq t \forall s \in F$  are also of considerable interest, but will be omitted here.) This class contains many interesting examples, some of them well known – like Gaussian processes, or more generally, processes with elliptically contoured finite dimensional distributions; other rather unexpected – like Kingman's example of discrete time martingale, which forms martingale difference system, when the time is reversed. The talk will review up-to-date results.

The interest will be in the properties determined by "higher order conditional structure," typically by

$$(2) \quad E(X_t^2 | X_s : s \in F), \text{ where } F \subset T \text{ is finite.}$$

Finiteness of second moments alone is known to be a severe restriction on the possible values of coefficients  $\{a_s\}$  in (1), and in many situations knowing the form of second-order structure (2) provides surprisingly accurate information about the process  $(X_t)_{t \in T}$ .

# ON SOME INEQUALITIES OF SEMIMARTINGALES

by

Ching-Sung Chou

National Central University  
Chung-Li, Taiwan

## ABSTRACT

Let  $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, P)$  be a probability space satisfying the usual conditions,  $L : (\Omega, \mathcal{F}) \rightarrow (\mathbb{R}, \beta(\mathbb{R}_+))$  be a positive random variable (not necessary a  $(\mathcal{F}_t)$ -stopping time), and let  $Z_t = e[1_{(L > t)} | \mathcal{F}_t]$ , it is easy to see that  $Z$  is a potential of class (D). Thus we can consider the Doob-Meyer decomposition of  $Z$ , i.e.  $Z = M - A$ . All processes  $(X_t)$  considered in the following are càdlàg and  $X_0 = 0$ .

**Lemma 1.** Let  $X$  and  $Y$  be the semimartingales, we suppose that the r.v.  $\langle Y, Y \rangle_\infty$  is finite, denote

$$H_S = E[\langle Y, Y \rangle_\infty - \langle Y, Y \rangle_S | \mathcal{F}_{S-}] \quad \text{for predictable stopping time } S,$$

then, we have

$$E \left[ \int_0^L |d\langle X, Y \rangle_s| \right] \leq c E \left[ \left( \int_0^L H_s^* - d\langle X, X \rangle_s \right)^{1/2} \right].$$

**Lemma 2.** With the same notations as lemma 1, if  $Y = (\frac{1}{Z_-} \cdot M)^L$ , then, we have

$$E \left[ \int_0^L |d\langle X, Y \rangle_s| \right] \leq c E \left[ \left( 1 + \log \frac{1}{I_L} \right)^{1/2} \langle X, X \rangle_L^{1/2} \right]$$

where  $I_L = \inf_{0 \leq s \leq L} Z_{s-}$ .

**Theorem.** Let  $\Phi$  be a convex moderate function. Then there exists a constant  $C_\Phi$ , depending only on  $\Phi$ , such that for all local martingale  $X$  with  $\langle X, X \rangle$  exists, we have

$$E \left[ \Phi \left( |X, X|_L^{1/2} \right) \right] \leq C_\Phi E \left[ \Phi \left( \left( 1 + \log \frac{1}{I_L} \right)^{1/2} \left( X_L^* + \langle X, X \rangle_L^{1/2} \right) \right) \right]$$

and

$$E \left[ \Phi(X_L^*) \right] \leq C_\Phi E \left[ \Phi \left( \left( 1 + \log \frac{1}{I_L} \right)^{1/2} \left( |X, X|_L^{1/2} + \langle X, X \rangle_L^{1/2} \right) \right) \right].$$

# ON THE CONVERGENCE RATE OF ANNEALING PROCESSES\*

by

Tzuu-Shuh Chiang

and

Yunshyong Chow

*Institute of Mathematics  
Taipei, Taiwan*

## ABSTRACT

For the class of inhomogeneous Markov processes arising from simulated annealing, we show that

$$\lim_{t \rightarrow \infty} P(X_t = i) / \exp(-u(i)/T(t))$$

exists and is positive for each state  $i$ , where  $T(t)$  is the temperature at time  $t$  and  $u(i)$  the energy level at state  $i$  (we assume  $\min_i u(i) = 0$ ). Our method is to consider the forward equations associated with such Markov processes.

**Keywords and phrases:** Simulated annealing, forward equations, Perron-Frobenius theorem, convergence rate.

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\* AMS 1980 subject classifications: Primary 60J27, 60J99, Secondary 15A51, 15A18, 90B40.

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# ON THE GROWTH OF THE MULTITYPE SUPERCRITICAL BRANCHING PROCESS IN A RANDOM ENVIRONMENT

by

Harry Cohn

*The University of Melbourne  
Melbourne, Australia*

## ABSTRACT

Let  $\{Z_n\}$  be a Athreya-Karlin multitype branching process in a random environment. If the environmental distributions have first and second order moments bounded away from  $\infty$ , then norming the  $\{Z_n\}$ 's components by the expectations yields a  $L^2$ -convergent process. The proof is based on a martingale-subsequence approach which reduces the study to the properties of i.i.d. random variables. The Furstenberg-Kesten a.s. convergence theorem for random matrices yields stochastic compacticity. Unlike the Galton-Watson case the notion of eigenvalue does not appear in our arguments.

# ASYMPTOTICS FOR FINITE PARTICLE SYSTEMS

by

Ted Cox

*Syracuse University  
Syracuse, NY*

## ABSTRACT

Infinite particle systems are stochastic processes which model the behavior of large systems of stochastically interacting components. Typically the components are located at the points of a set  $\Lambda \subset \mathbb{Z}^d$ , and can be in several different states. From the applied point of view one is interested in the behavior of such processes when  $\Lambda$  is finite but very large. The usual approach is to replace  $\Lambda$  with  $\mathbb{Z}^d$ , and then study the infinite system. This leads to a rich and beautiful theory (see *Interacting Particle Systems* by Liggett), and it is generally believed that infinite systems provide good approximations for large finite systems.

But the temporal behavior of finite systems is very different from that of infinite systems. Interesting infinite systems such as critical branching random walk, the voter model, and the contact process enjoy some type of critical behavior (phase transition) or have multiple equilibria, while the corresponding finite systems do not; in fact they eventually reach traps.

The problem considered here is to try to make more precise the meaning of "infinite systems approximate large finite systems", at least for the three processes mentioned above. To do so we consider finite systems  $\eta_t^{(N)}$  defined on the torus  $\Lambda(N)$  of side  $N$  in  $\mathbb{Z}^d$  and discuss two specific points. The first is to determine the asymptotics of trapping times  $\tau^{(N)}$ , i.e. to find a (nonrandom) sequence  $S_N$  such that  $\tau^{(N)} \approx S_N$  in some sense as  $N \rightarrow \infty$  (this is easy to do for critical branching, but not for the other models). The second is to describe (locally) the law of  $\eta_{t_N}^{(N)}$  as  $N \rightarrow \infty$  for times  $t_N$  which are of the same (smaller, or larger) order as  $S_N$ , and to relate this to properties (especially equilibrium states) of the infinite system  $\eta_t$ .

# AN APPROXIMATION OF STOPPED SUMS WITH APPLICATIONS IN QUEUEING THEORY

by

Miklós Csörgő\*

*Carleton University  
Ottawa, Ontario, Canada*

Paul Deheuvels

*Université Paris VI  
Paris, France*

Lajos Horváth\*\*\*

*Szeged University, Bolyai Institute  
Aradi Vétanúk, Hungary*

## ABSTRACT

We prove strong approximations for partial sums indexed by a renewal process. The obtained results are optimal. The established probability inequalities are also used to get bounds for the rate of convergence of some limit theorems in queueing theory.

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# BOUNDARY LOCAL TIME AND THE ANALYSIS OF SMALL PARAMETER EXIT PROBLEMS WITH CHARACTERISTIC BOUNDARIES

by

Martin V. Day

*Virginia Tech  
Blacksburg, VA*

## ABSTRACT

The subject of this talk is a variant of the "exit problem" from Freidlin and Wentzell's study of small noise diffusions. The problem is to determine (in the small noise limit:  $\epsilon \rightarrow 0$ ) the distribution of the first exit point from a region  $D$  for a diffusion process whose drift components constitute a "stable" dynamical system in  $D$  and whose diffusion coefficients are multiplied by the small parameter  $\epsilon$ . In the case discussed by Freidlin and Wentzell the drift is assumed to enter  $D$  non-tangentially across its boundary. However in many applications  $D$  is such that the drift is tangential to the boundary. In this talk we will explain the connection between this problem and the asymptotics of the equilibrium density  $p^*(x)$  of the small noise diffusion subject to normal reflection off the boundary of  $D$ . The analogous connection is at the heart of recent advances for the non-tangential case. The tangential case is especially interesting because of the key role played by the local time and boundary processes associated with the reflected diffusion.

SOME THEOREMS OF CENTRAL LIMIT TYPE FOR MARKOV  
PATHS AND SOME PROPERTIES OF GAUSSIAN  
RANDOM FIELDS

by

Raya Epstein

*Technion-Israel Institute of Technology  
Haifa, Israel*

and

*University of California  
Berkeley, CA*

ABSTRACT

Our primary aim is to "build" versions of generalized Gaussian processes from simple, elementary components in such a way that as many as possible of the esoteric properties of these elusive objects become intuitive. For generalized Gaussian processes, or fields, indexed by smooth functions or measures of  $\mathbb{R}^d$ , our building blocks will be simple Markov processes whose state space is  $\mathbb{R}^d$ . Roughly speaking, by summing functions of the local times of the Markov processes we shall, via a central limit theorem type of result, obtain the Gaussian field.

This central limit result, together with related results indicating how additive functionals of the Markov processes generate additive functionals of the fields, and, indeed, how the entire Fock space or Wiener chaos structure of the  $L^2$  space of a Gaussian field can be identified with additive functionals of Markov processes, yield considerable insight into properties of generalized Gaussian processes such as Markovianess, self-similarity, "locality" or functionals, etc.



# ON A RANDOM WALK WITH INTERNAL DEGREES OF FREEDOM

G. I. Falin

*Moscow State University*

## ABSTRACT

Let  $\xi_t = (x_t, y_t)$ ,  $t \in Z_+$ , be a Markov chain with state space  $Z_+ \times \{1, \dots, N\}$  and transition probabilities  $p_{nm}^{ij} = P\{\xi_{t+1} = (j, m) | \xi_t = (i, n)\}$  such that: (a)  $p_{nm}^{ij} = p_{nm}^{i-j}$  for  $i \geq 1$ ; (b)  $p_{nm}^{\alpha i} = 0$  for  $i > L$ ; (c)  $p_{nm}^k = 0$  for  $|k| > 1$ . A major motivation for this chain derives from a specific technical concept used in modern communication systems (hybrid switching with movable voice/data boundary, asynchronous data interpolation in analogue speech, etc.)

Denote by  $P^{\alpha i}, P^{+1}, P^0, P^{-1}$  the  $N \times N$  matrices with components  $p_{nm}^{\alpha i}, p_{nm}^{+1}, p_{nm}^0, p_{nm}^{-1}$ ;  $P = P^{+1} + P^0 + P^{-1}$ ;  $M_n = \sum_{m=1}^N (p_{nm}^{+1} - p_{nm}^{-1}) = p_n^{+1} - p_n^{-1}$ ;  $\pi = (\pi_1, \dots, \pi_N)$  is the vector of stationary probabilities corresponding to the (irreducible and aperiodic) Markov chain with transition probabilities  $p_{nm} = p_{nm}^{+1} + p_{nm}^0 + p_{nm}^{-1}$ ;  $A$  is  $(N-1) \times (N-1)$  matrix with components  $p_{nm}, 1 \leq n, m \leq N-1$ ;  $\pi_{in} = \lim_{t \rightarrow \infty} P\{\xi_t = (i, n)\}$ ;  $[a_n]_{1 \leq n \leq N-1}$  is the vector  $(a_1, \dots, a_{N-1})$ .

**Theorem 1.** The chain  $\xi_t$  is ergodic, positive recurrent or transient iff  $\sum_{n=1}^N \pi_n M_n$  is negative, equal to zero or positive accordingly.

The proof uses the well-known general ergodicity, recurrence and transience criteria based on mean drift. The test function  $\varphi$  looks as follows:  $\varphi(i, n) = \text{Const} + a_n i$ , where  $a_N = 0, (a_1, \dots, a_{N-1})^T = -A^{-1}\{(M_1, \dots, M_{N-1})^T - \theta \cdot \sum_{n=1}^N \pi_n M_n / (1 - \pi_N) \cdot (1, \dots, 1)^T\}, 0 < \theta < 1$ .

**Theorem 2.** Let  $\epsilon = -\sum_{n=1}^N \pi_n M_n \rightarrow 0+$  in such a manner: (a)  $P = \sum_{i=0}^L P^{\alpha i}$ ; (b)  $P$  does not change; (c)  $-\sum_{m=1}^N \pi_{om} \left[ p_m^{-1} - \sum_{i=1}^L \frac{i(i-1)}{2} p_m^{\alpha i} \right] + \left[ \sum_{m=1}^N \pi_{om} (p_m^{+1} - p_m^{-1} - \sum_{i=1}^L i p_m^{\alpha i}) \right]_{1 \leq n \leq N-1} \cdot A^{-1} \cdot (M_1, \dots, M_{N-1})^T \rightarrow 0$ . Then under steady state  $E \exp(-s \epsilon x_t; y_t = n) \rightarrow \pi_n / (1 + \mu s)$ , where

$$\mu = \sum_{m=1}^N \pi_m p_m^{-1} - \left[ \sum_{m=1}^N \pi_m (p_m^{+1} - p_m^{-1}) \right]_{1 \leq n \leq N-1} \cdot A^{-1} \cdot (M_1, \dots, M_{N-1})^T.$$

# THE IMPACT OF THE COMPOSITION OF THE CUSTOMER BASE IN GENERAL QUEUEING MODELS

by

A. Federgruen

*Columbia University  
New York, NY*

and

H. Groenvelt

*University of Rochester  
Rochester, NY*

## ABSTRACT

We consider general queueing models dealing with multiple classes of customers and address the question under what conditions and in what (stochastic) sense the marginal increase in various performance measures, resulting from the addition of a new class of customers to an existing system, is larger than if the same class were added to a system dealing with only a subset of its current customer base.

Our results enhance our understanding of the dependence of various performance measures with respect to the composition of the customer base. In addition they translate readily into convexity results in an (appropriately defined) arrival rate.

# REACTION-DIFFUSION EQUATIONS WITH SMALL PARAMETER: PROBABILISTIC APPROACH

by

Mark Freidlin

*University of Maryland  
College Park, MD*

## ABSTRACT

We consider the system of PDE

$$\frac{\partial u_k(t, x)}{\partial t} = L_k u_k + f_k(x; u_1, \dots, u_n), \quad (*)$$
$$x \in \mathbb{R}^r, \quad k = 1, \dots, n,$$

where  $L_k$  are second-order elliptic, may be degenerate operators. If  $f_k$  are linear in  $u_1, \dots, u_n$ , then a Markov process  $(X_t, \nu_t)$  in the state space  $\mathbb{R}^r \times \{1, \dots, n\}$  connected with system  $(*)$  and  $u_k(T, x)$  can be represented as the expectation of proper functionals of the process. In the non-linear case this representation gives an integral equation for the solution of problem  $(*)$ . If there is a small parameter  $\varepsilon$  in equation  $(*)$ , then the limiting behavior of the solution is defined by the limit theorems for the corresponding family of processes  $(X_t^\varepsilon, \nu_t^\varepsilon)$  as  $\varepsilon \downarrow 0$ .

We study the limit theorems for the family  $(X_t^\varepsilon, \nu_t^\varepsilon)$  and deduce from them some results on the behavior of  $u_k^\varepsilon(t, x)$  as  $\varepsilon \downarrow 0$ .

# DEGENERATE BELLMAN EQUATION AND ITS APPLICATIONS

by

**Masatoshi Fujisaki**

*Kobe University of Commerce*

and

*University of North Carolina  
Chapel Hill, NC*

## ABSTRACT

Consider the following Bellman equation with degenerate diffusion coefficients:

$$(1) \quad \inf_{\alpha \in A} \left\{ \partial_s v + \sum_{1 \leq i, j \leq \nu} a_{ij}(\alpha, s, x) \partial_i \partial_j v + \sum_{i=1}^d b_i(\alpha, s, x) \partial_i v - c(\alpha, s, x) v + f(\alpha, s, x) \right\} = 0, \quad 0 \leq s < T, \quad x \in R^d (d \geq 2),$$

$$v(T, x) = g(x), \quad x \in R^d,$$

where  $1 \leq \nu < d$ ,  $a = (a_{ij})$ ,  $1 \leq i, j \leq \nu$ , is a positive definite matrix, and  $A$  is a separable metric space. Under suitable conditions of the coefficients about regularities and boundedness, we can show the existence and uniqueness of generalized solution of (1) ([1]).

It is also interesting to consider the case where the coefficients are not bounded with respect to the parameter  $\alpha$ . Some extensions of [1] and [2], and their applications to differential equations will be presented in the talk.

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## PROBLEMS IN THE SILTING OF DAMS

by

J. Gani

*University of California  
Santa Barbara, CA*

### ABSTRACT

In the classical theory of dams, the content  $Z_t$  of a finite dam of capacity  $K$  at time  $t = 0, 1, 2, \dots$ , subject to i.i.d. inputs  $X_t$  in  $(t, t+1)$  and a release  $M$  (or  $Z_t + X_t$  if it is less than  $M$ ) at  $t+1$  is given by

$$Z_{t+1} = \min\{Z_t + X_t, K\} - \min\{Z_t + X_t, M\},$$

where  $\{Z_t\}$  forms a Markov chain.

A practical problem which arises in many dams is silting due to the sedimentation of particles from the water input into the dam. If  $Y_t$  denotes the amount of silt deposited in the dam in  $(t, t+1)$ , and the  $\{Y_t\}$  are i.i.d., then  $S_{t+1} = \sum_{n=0}^t Y_n$  is the total silt in the dam at time  $t+1$ , and its real capacity is  $K_{t+1} = K - S_{t+1}$ , a Markov chain. The real content of the dam is now

$$U_{t+1} = \min\{U_t + X_t, K_{t+1}\} - \min\{U_t + X_t, M\},$$

where  $U_t = Z_t - S_t$ , and  $\{U_t, K_t\}$  forms a bivariate Markov chain whether the  $\{X_t\}, \{Y_t\}$  are independent of each other or not. A practical theory of dams with silting can be developed on these assumptions.

## RECENT RESULTS ON QUEUES WITH TIME VARIATIONS

by

E. Gelenbe, S. Lefebvre, J. M. Vincent

*Laboratoire ISEM  
Université de Paris Sud  
Orsay, France*

### ABSTRACT

In this paper we shall discuss some recent results concerning two approaches to modelling queueing systems with time variations in the context of specific applications.

We shall first examine a special type of queueing system called the resequencing queue. Here, the service mechanism is based on reconstituting the arrival sequence at the server by compensating for a random disordering delay which has been introduced before the service mechanism. This problem is of importance in communication networks and in distributed data base systems, where variations in traffic patterns with time are often present. The analysis available so far only assumed time independent arrival sequences. We provide analytical results for the time dependent Poisson arrival model, as well as for a slowly varying Markovian arrival process.

We then proceed to the case of queueing networks which may have time-of-day type variations in their arrival and service parameters, and provide analytical and numerical approximations to their stationary behaviour, and compare these approximations to the 'naive' approximations which may be used.

# THE ROLE OF POISSON'S EQUATION IN STEADY-STATE SIMULATION OUTPUT ANALYSIS

by

Peter W. Glynn

*University of Wisconsin  
Mason, WI*

## ABSTRACT

Given a Markov chain with transition kernel  $P$ , Poisson's equation (for functions) involves solving  $(I - P)g = f$  for the unknown  $g$ ; the analogue for measures requires finding  $\nu$ , where  $\nu(I - P) = \eta$ . In this talk, we will focus on the key role that Poisson's equation plays in the analysis of several important problems related to steady-state output analysis. In particular, suppose  $P$  is positive recurrent with invariant probability  $\pi$  and that one wishes to compute  $\pi f$  via simulation. A significant literature in simulation is devoted to studying the variability and the bias of the sample mean, as an estimator of  $\pi f$ ; the solution  $g$  to  $(I - P)g = f - \pi f$  (and its related martingale structure) provides valuable insight into both of these characteristics of the sample mean. The measure version of Poisson's equation arises in the Monte Carlo calculation of the derivative of  $\alpha(\theta)$ , where  $\alpha(\theta) = \pi(\theta)f$  and  $\pi(\theta)$  is the invariant probability of a transition kernel  $P(\theta)$  depending on the real-values parameter  $\theta$ . These derivative estimators play an important role in both the optimization and sensitivity analysis of complex stochastic systems.

# LIMIT FLUCTUATIONS OF A CRITICAL BRANCHING PARTICLE SYSTEM IN A RANDOM MEDIUM

by

Donald A. Dawson

*Carleton University, Canada*

Klaus Fleischmann

*Academy of Sciences of G.D.R.*

Luis G. Gorostiza

*Centro de Investigación y Estudios Avanzados, Mexico*

## ABSTRACT

We consider a particle system in  $R^d$  where the particles are subject to spatial motion according to a symmetric stable law and to a critical branching law in the domain of attraction of a stable law. The branching intensity is given by a realization of an ergodic random medium. We show that under natural assumptions the limit fluctuations of the appropriately scaled system around the macroscopic flow are the same as those given by the averaged medium. The (hydrodynamic) limit fluctuations process is an  $S'(R^d)$  - valued Ornstein - Uhlenbeck process. The Langevin equation for this process is of the form

$$dY_t = -(-\Delta)^{\alpha/2} Y_t dt + dZ_t,$$

where  $Z$  is an  $S'(R^d)$  - valued process with independent increments which are distributed according to an asymmetric stable law. The convergence proof involves an analysis of a nonlinear integral equation with random coefficients.



# DOMAINS OF ATTRACTION FOR A FAMILY OF PROCESSES BETWEEN SUPREMA AND SUMS

by

Priscilla Greenwood

and

Gerard Hooghiemstra

## ABSTRACT

Let  $\{Y_i, i = 0, 1, \dots\}$  be an i.i.d. sequence with distribution  $F$ . Define a sequence  $S_n$  by  $S_0 = Y_0(1 - \gamma)^{-1}$ ,  $S_n = \max(S_{n-1}, \gamma S_{n-1} + Y_n)$  where  $0 \leq \gamma < 1$ . Classes of  $F$  are identified for which the sequence  $S_n$ , suitably normalized, converges weakly. Three possible limit distributions are described. The weak limits of the associated random functions in  $D(0, \infty)$  are non-decreasing Markov jump processes which form an intermediary class between extremal processes and non-decreasing stable processes. They also generate new examples of so-called self-similar processes.

# LOCAL BEHAVIOUR OF SIMPLE STOCHASTIC MODELS

by

Rudolf Grübel

## ABSTRACT

Consider a  $\{0,1\}$ -valued process modelling a simple repair system: 0 indicates working order, 1 repair. Life times and repair times are independent and identically distributed respectively, we start with an unused component at time  $t = 0$ . Interest is in the probability  $p(t)$  of being in repair state at time  $t$ , life time and repair time distributions are regarded as known.

This is a typical applied probability situation: classical theory deals with asymptotic behaviour of  $p(t)$  as  $t \rightarrow \infty$ ; known cases, i.e., where  $p$  can be given explicitly, are very rare.

We regard such simple models as functions relating the quantity of interest to some given input quantities. In a variety of models this function admits a Taylor expansion about a known case. The resulting approximation often displays relevant features of the model which are inaccessible by asymptotic theory.

# RESPONSE TIMES IN M/M/1 TIME SHARING SCHEMES WITH LIMITED NUMBER OF SERVICE POSITIONS

by

Benjamin Avi Itzhak\*

and

Shlomo Halfin

*Bell Communications Research*

## ABSTRACT

Two service schemes for an M/M/1 time sharing system with a limited number of service positions are studied. Both schemes possess the equilibrium properties of symmetric queues, however in the first one, a preempted job is placed at the end of the waiting line; while in the second one, it is placed at the head of the line. Methods for calculating the Laplace transforms and moments of the response times are presented. The variances of the response times are then compared numerically to indicate that the first scheme is superior to the second scheme. It is also indicated that in both cases the response time variance decreases when the number of service positions increase.

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\* Presently with RUTCOR, Rutgers Center for Operations Research

# WEAK CONVERGENCE OF THE WEIGHTED EMPIRICAL PROCESS INDEXED BY RECTANGLES UNDER MIXING CONDITION

by

Michel Harel

*Institut Universitaire de Technologie  
de Limoges, France*

and

Madan Puri

*Indiana University  
Bloomington, IN*

## ABSTRACT

After the results of Einmahl, Ruymgaart and Wellner (1) on the convergence of the weighted multivariate empirical processes indexed by rectangles in the independent case, we generalize the results of Harel (2) on the convergence of the empirical processes indexed by points to the empirical processes indexed by rectangles under  $\varphi$  mixing or strong mixing condition. We introduce a Skorohod topology on some space of functions defined on the set of rectangles of  $[0, 1]^k$  and after that we prove the weak convergence of those processes with respect to the Skorohod topology.

**Keywords:** empirical process indexed by rectangles, weight function, Skorohod topology,  $\varphi$  mixing, strong mixing.

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# RANDOM WALK ON THE QUADRANT

by

L. A. Klein Haneveld

*University of Amsterdam*

## ABSTRACT

Consider discrete time random walk on the two-dimensional lattice  $\mathbb{Z}^2$ . The walk is assumed to be skipfree, i.e. jumps are possible from a point to itself and its nearest eight neighbours only. The walk is started at an arbitrary point with nonnegative coordinates, and killed as soon as one of the coordinates is negative. Let the stochastic vector  $X = (X_1, X_2)$  have the probability distribution of the step. Also, let  $T(i)$  be the duration (i.e. the total number of steps until death) of the walk starting at the point  $i = (i_1, i_2)$ . Concerning  $T(i)$  the following simple result has been obtained.

**Theorem.** Let the walk be driftless, i.e.  $EX_1 = 0 = EX_2$ . Then the expected duration of the walk is given by

$$ET(i) = \begin{cases} \frac{(i_1+1)(i_2+1)}{-EX_1X_2} & \text{if } EX_1X_2 < 0, \\ \infty & \text{if } EX_1X_2 \geq 0. \end{cases}$$

This theorem can be derived from more general results concerning the generating function  $P(x, y, s)$  of the transition probabilities of the stopped random walk. The generating function is defined as follows. If  $p(i, j, n)$  is the probability, starting at  $i$ , to arrive before death at  $j$  in  $n$  steps, then

$$P(x, y, s) := \sum_{i, j, n} x^i y^j s^n p(i, j, n), \quad (\text{with } x^i := x_1^{i_1} x_2^{i_2} \text{ and similarly for } y^j)$$

where  $x, y \in \mathcal{C}^2$ ,  $s \in \mathcal{C}$ ,  $i, j \in \mathbb{Z}^2$ ,  $n \in \mathbb{Z}$ , and the summation is over  $i_1, i_2, j_1, j_2, n \geq 0$ . This power series is known to converge if

$$|x_1|, |x_2| > 1, \quad |y_1|, |y_2| \leq 1, \quad |s| < 1.$$

Under the assumption that the distribution of  $X$  is not confined to some closed half plane with the origin on its boundary, for  $P(x, y, s)$  an expression in closed form has been obtained, by means of an analytical method due to J. Groeneveld. In addition the question whether  $P(x, y, 1)$  converges has been investigated, and expressions for  $P(x, y, 1)$  have been found, by taking the limit for  $s \rightarrow 1$  and using a stochastic monotonicity argument. In the case of driftless random walk this leads to the above theorem.

# BROWNIAN MODELS OF OPEN QUEUEING NETWORKS\*

by

J. M. Harrison

*Stanford University  
Stanford, CA*

## ABSTRACT

We consider a class of multidimensional diffusion processes that arise as heavy traffic approximations for open queueing networks. It is explained in concrete terms how one approximates a conventional queueing model by one of these Brownian system models, and some basic properties of such Brownian models are discussed. This is largely a recapitulation of earlier work on heavy traffic limit theorems, with the emphasis placed on modeling intuition.

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\* Based on joint work with R. J. Williams

# INSENSITIVITY WITH INTERRUPTIONS

by

W. Henderson

and

P. Taylor

*University of Adelaide  
Australia*

## ABSTRACT

The theory of insensitivity within Generalised Semi-Markov Schemes is extended to cover classes of models in which the generally distributed lifetimes can be terminated prematurely by the deaths of negative exponentially distributed lifetimes. As a consequence of this approach it is shown that there exists classes of processes which are insensitive with respect to characteristics of the general distributions other than the mean. A variety of examples is given. One models the interaction between bushfires and vegetation in remote forest regions. The others analyse queues in which the generally distributed service times are interrupted in either an abort or a restart fashion.

ON CRITERIA OF OPTIMALITY IN ESTIMATION  
FOR STOCHASTIC PROCESSES

by

C. C. Heyde

*Australian National University  
Canberra, Australia*

ABSTRACT

Optimality is a widely and loosely used term in inference for stochastic procedures. This talk will be concerned with categorizing and contrasting various criteria which give the "best" estimator within a specified class of alternatives.



# A LIMIT THEOREM FOR RANDOM WALK IN RANDOM SCENERY

by

W. Th. F. den Hollander

*Delft University of Technology  
Netherlands*

## ABSTRACT

The following problem will be discussed. Consider the lattice  $Z^d, d \geq 1$ . The points are colored black and white randomly (stationary and ergodic). On the lattice a random walk takes place that is independent of the coloring. Under what conditions on coloring and walk do the following mixing properties hold?

$$(1) \quad \lim_{n \rightarrow \infty} P([s]_0 \cap [t]_n) = P([s]_0)P([t]_0),$$

$$(2) \quad \lim_{k \rightarrow \infty} P([s]_0 \cap [t]_{T_k}) = P([s]_0)P([t]_0 | 0 \text{ is black}).$$

Here:

$P$  probability measure for combined process of coloring and walk,

$s, t$  local color sceneries of 0 (the origin),

$[s]_n$  event that walker sees local scenery  $s$  relative to his position at time  $n$ ,

$T_k$  time at which walker hits a black site for the  $k$ th time.

(2) cannot be deduced from (1) and is in fact a much deeper property than (1). An interesting consequence of (2) is that  $T_{k+1} - T_k$  has a limit distribution as  $k \rightarrow \infty$ . This limit distribution can be expressed in terms of the distribution of  $T_1$ .

# RANDOM MEASURES AND PARTICLE STATISTICS

by

Joseph Horowitz

*University of Massachusetts  
Amherst, MA*

## ABSTRACT

A model based on random measures is given for the distribution of weights of samples from a fine particle system (e.g., a powder) involving either a finite or an infinite number of particles, and including a possible continuum component; the model allows for breakdown or agglomeration of particles during the sampling process, i.e., a kind of "uncertainty principle". Applying the model to a random powder whose particle masses are given by the jump sizes of a subordinator, we have the following results: in the case of a stable subordinator, for almost every realization of such a powder, the weight distribution is absolutely continuous and has a smooth density; when the subordinator is a gamma process, the weight distribution of samples is purely singular if the "sampling fraction" (by volume) is sufficiently close to zero or one, contrary to the usual assertion that weight distributions are normal.

STATISTICAL ALTERNATIVES FOR MEASURING  
FUNCTIONAL CONCORDANCE BETWEEN  
CEREBRAL HEMISPHERES

by

A. de Hoyos\*

*University de Campinas*

N. D. Cook

*University of Oxford*

J. Grinberg\*\*

*University of Mexico*

ABSTRACT

According to the experimental results obtained by one of the authors interhemispheric concordance is not only related to individual mental states but also to levels of communication between individuals. Several statistical tests were developed and compared for evaluation of changes in patterns of interhemispheric concordance, under a simple non-verbal communication experiment.

**Keywords:** EEG statistical analysis, interhemispheric concordance, non-verbal communication, ANOVA for time series.

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# ON SOME CHARACTERIZATIONS OF THE POISSON PROCESSES

by

**Wen-Jang Huang**

*National Sun Yat-sen University  
Kaohsiung, Taiwan, R.O.C.*

## ABSTRACT

We give some characterizations of Poisson processes about thinning of point processes that each arrival may be split into two new atoms. More precisely, we first allow the thinning probabilities be time-dependent, and obtain a joint characterization of the Bernoulli distribution and the Poisson process. Then under the situation that the thinning probabilities be independent of time, we show that thinning of arbitrarily delayed renewal processes produces uncorrelated thinned processes if and only if the renewal process is Poisson, which is a generalization of Chandramohan and Liang (1985). Some other related characterization results are also studied.

# NORMAL APPROXIMATION IN AN URN MODEL WITH INDISTINGUISHABLE BALLS

by

N. K. Indira

*Indian Statistical Institute  
Bangalore, India*

and

V. V. Menon

*Banaras Hindu University  
Varanasi, India*

## ABSTRACT

For the Bose-Einstein Statistics, where  $n$  indistinguishable balls are distributed in  $m$  urns such that all the arrangements are equally likely, define the random variables.

$M_k$  = number of urns containing exactly  $k$  balls each;

$N_k$  = number of urns containing at least  $k$  balls each.

We consider the approximation of the distributions of  $M_k$  and  $N_k$  by suitable normal distributions, for large but finite  $m$ . Estimates are found for the error in the approximation to both the probability function and the distribution function in each case. These results apply also to the alternative model where no urn is allowed to be empty. The results are illustrated by some numerical examples.

# ON ESTIMATING MEASURES OF PERFORMANCE FOR QUEUES AND SURVIVAL MODELS

by

D. P. Gaver

and

P. A. Jacobs

*Naval Postgraduate School  
Monterey, CA*

## ABSTRACT

Many random times of interest,  $T$ , in queueing and reliability models have survivor functions which are asymptotically exponential; that is,  $P\{T > t\} \sim ce^{-\kappa t}$  as  $t \rightarrow \infty$ . Often the parameter  $\kappa$  is the solution to an equation involving the transforms or moment generating functions of the component distributions in the model. The parameter  $c$  is often a function of  $\kappa$  and the component distributions. Suppose that finitely many observations are all that is known about some or all of the component distributions of the model. If the parametric forms of the component distributions are known, then estimates of the parameters of the distributions can be found via maximum likelihood or method of moments etc., and  $\kappa$  and  $c$  can be determined parametrically. However, if, perhaps because of small sample sizes, the parametric forms of the distribution functions are incorrectly specified, then parametric estimates of  $\kappa$  and  $c$  can be very misleading. The nonparametric estimation of  $\kappa$  and  $c$  will be discussed. The methodology will be illustrated in a number of specific models.

# BRANCHING PROCESSES AS MARKOV FIELDS

by

Peter Jagers

*Chalmers University  
Gothenburg*

## ABSTRACT

It is hard to find a natural state space that renders realistic models of population development Markovian in time. Galton-Watson or birth-and-death style processes, considering only population size, are obviously over simplified. But even the complete age structure at a given time may be insufficient for independence of the past. It turns out that the difficulty is not state space but rather "time": Viewed as a process on the set of possible individuals, partially ordered by descent, even very general populations models has a Markovian structure (inheritance from mother and then you lead or own life).

This idea is used to formulate general branching processes in abstract type spaces as Markov fields. The strong Markov branching property holds and an intrinsic martingale (with partially ordered indices, which are set of individuals) is exhibited. The martingale is uniformly integrable under the famed 'xlogx' - condition. Together with Markov renewal theory and classical limit theory for sums of independent random variables it catches the asymptotics of the process, as real time passes.

# SPACE-TIME STOCHASTIC PROCESSES

by

Dudley Paul Johnson

*University of Calgary  
Canada*

## ABSTRACT

In conducting a scientific experiment, one gets a sequence of measurements  $[X(n), T(n)]$  where  $X(n)$  is a random variable representing the  $n$ th measured spatial change in a system since the last measurement was taken and  $T(n)$  is a random variable representing the measured time since the last measurement was taken.

Using the theory of algebraic representations of discrete time stochastic processes in Johnson [1, 2, 3], we describe: 1) how to estimate the algebraic representation of the general space-time stochastic process from a sample  $X(1), T(1), \dots, X(n), T(n)$ , 2) how to use the estimated algebraic representation to compute the behavior of the system when the times between observations happen to be so small that the system was effectively observed continuously, 3) how space-time models allow for the act of observing the system to affect the system itself and 4) the close relationship between space-time stochastic processes and quantum stochastic processes.

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OPTIMAL PORTFOLIO AND CONSUMPTION DECISIONS FOR  
A "SMALL INVESTOR" ON A FINITE HORIZON\*

by

Ioannis Karatzas

*Columbia University  
New York, NY*

John P. Lehoczky

and

Steven E. Shreve

*Carnegie-Mellon University  
Pittsburgh, PA*

ABSTRACT

A general consumption/investment problem is considered for an agent whose actions cannot affect the market prices, and who strives to maximize total expected discounted utility of both consumption and terminal wealth. Under very general conditions on the nature of the market model and on the utility functions of the agent, it is shown how to approach the above problem by considering separately the two, more elementary ones of maximizing utility of consumption only and of maximizing utility of terminal wealth only, and then appropriately composing them. The optimal consumption and wealth processes are obtained quite explicitly. In the case of a market model with constant coefficients, the optimal portfolio and consumption rules are derived very explicitly in feedback form (on the current level of wealth.)

**Keywords:** Portfolio and consumption processes, utility functions, stochastic control, martingale representation theorems, change of probability measure, Feynman-Kac theorem.

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# RANDOM TIME CHANGES FOR PROCESSES WITH RANDOM BIRTH AND DEATH

by

H. Kaspí

*Technion  
Haifa, Israel*

## ABSTRACT

During recent years a great deal of work has been done on Markov processes  $(Y_t, Q)$  with random birth and death times (denoted  $\alpha$  and  $\beta$  respectively). We study time changes for such processes. An important class of those processes are stationary processes, where  $Q$  is invariant under the time shift operator. In this case, the 1-dimensional distribution is excessive relative to the transition semigroup.

In the classical case of a Markov process  $(X_t, P), t \in [0, \zeta)$ , with a stationary transition function, a time change consists of an increasing process  $(S_t)$ , so that  $\tilde{X}_t = X_{S_t}$  is a Markov process (with a stationary transition function).  $(S_t)$  is obtained as an inverse of a continuous additive functional  $(A_t)_{t \geq 0}$ .

In the present situation, the additive functional, of the classical case, is replaced by a homogeneous random measure  $B$ , naturally related to  $(A_t)$ . An increasing process  $(B_t)$  (clock) is obtained from  $B$ , by fixing  $B_u$  at a point  $u \in (\alpha, \beta)$ ,  $(C_t)$  — the right continuous inverse of  $(B_t)$  replaced  $(S_t)$  of the classical case. We consider only time changes that preserve both the stationarity and the Markov property. To obtain this, some care is required in the choice of  $B_u$ . The details of this construction will be discussed.

We show that the time changed process will have

$$\nu(C) = Q \int_{[0,1]} 1_C(Y_t) B(dt)$$

as 1-dimensional distribution, and  $\tilde{P}_t f(x) = E^x(f(X_{S_t}))$  as transition semigroup  $((S_t) = (A^{-1})_t$  as above).  $\nu$  will be invariant for  $(\tilde{P}_t)$  iff  $B(\alpha, t) = \infty$  for all  $t > \alpha$   $Q$  a.e., and purely excessive iff  $\lim_{t \downarrow \alpha} B(\alpha, t) = 0$ .  $Q$  a.e. If time permits, we shall apply the time change to the study of the entrance behavior of the process near  $\alpha$ .

# MODELLING LOSS NETWORKS

by

**Frank Kelly**

*University of Cambridge  
England*

## ABSTRACT

This talk will concern models of telephone and computer communication networks. Even the simplest such models raise interesting and difficult mathematical questions: we shall touch on problems involving central limit theorems, fixed point approximations, optimization results, catastrophes and long range order.

# NEW DEVELOPMENTS IN THE STATISTICAL THEORY OF SHAPE

by

David G. Kendall

*Cambridge  
United Kingdom*

## ABSTRACT

(i) The shape-space for three labelled points in the plane has been shown by the writer to be a 2-dimensional sphere of radius  $\frac{1}{2}$ . If  $K$  is a compact convex set with interior, and if  $A, B$  and  $C$  are iid uniform inside  $K$ , they determine a random triangle and so a shape-measure on the shape-space, usefully described by the density  $m(x, y)$  relative to the measure  $dx dy$ , where  $(x, y)$  are appropriate cartesian coordinates in a stereographic projection of the shape-space. C. G. Small found that  $m$  has a constant value on a certain "basic segment" in the  $(x, y)$ -plane. Le Hui-lin and the writer have now found  $m(x, y)$  for all polygons  $K$ . The exotic geometry behind this construction will be presented by slides.

(ii) There is a similar problem if the plane carrying  $A, B$  and  $C$  is replaced by a 2-sphere of radius  $R$ . When  $R$  approaches infinity, the solution to this problem approaches the ~~size-and~~ shape density appropriate to (i). Asymptotics for large  $R$  are needed for the analysis of nearly "collinear" quasar triplets. A sketch will be given of the work on this problem by T. K. Carne, Le Hui-lin and the writer.

(iii) The Delaunay tessellation generated by a 2- (or  $m$ -) dimensional Poisson process yields a collection of random triangles (simplexes) that is expensive to simulate directly. It will be shown how for moderate  $m$  (say less than 10) a tile-by-tile simulation is possible, thus avoiding the construction of the tessellations themselves. This is used to investigate many unsolved problems, such as "how many tiles pack together at a point", and "what can we say about the shape of a Poisson-Delaunay tile for given  $m$ , and especially for large  $m$ ".

# COUPLING AND THE NEUMANN HEAT KERNEL

by

Wilfrid S. Kendall

## ABSTRACT

Let  $D_1, D_2$  be two convex domains (with smooth boundary) with corresponding Neumann heat kernels  $\eta_1(x, y, t), \eta_2(x, y, t)$ . Chavel has conjectured that if

$$x, y \in D_1 \subset D_2$$

then the following monotonicity result holds for all  $t$ :

$$\eta_1(x, y, t) \geq \eta_2(x, y, t)$$

He has proved this in the special case when  $D_2$  is a ball centered at either  $x$  or  $y$ . (As Chavel says, the proof is simply integration by parts!)

By exploiting the connection of the Neumann kernel to reflecting Brownian motion it is possible to prove the result in another special case; when  $D_1$  is a ball centered at either  $x$  or  $y$ . The proof depends on a careful coupling construction of the reflecting Brownian motions for  $D_1$  and for  $D_2$ , using the same probability space for both processes.

# PROPHET INEQUALITIES AND RELATED PROBLEMS OF OPTIMAL STOPPING

by

D. P. Kennedy

*University of Cambridge  
England*

## ABSTRACT

Prophet inequalities relate the expected maximum of a sequence of random variables (the average reward to a "prophet" with complete foresight who is allowed to choose one from the sequence) to the maximum expected reward of a gambler who chooses one from the sequence using (non-anticipative) stopping times. The best-known such result (due to Krengel, Sucheston and Garling [2]) shows that for a sequence of non-negative independent random variables the ratio of the return of the prophet to that of the gambler is always bounded by 2. Hill and Kertz [1] showed that for independent random variables taking values in  $[0, 1]$  the difference between the respective returns is bounded by  $1/4$ . A review will be presented of work extending these results to situations (i) where the players are permitted more than one choice from the sequence, and (ii) where the players have more than one sequence from which to sample. In addition, limiting results for threshold-stopped random variables related to extreme-value theory for independent sequences will be discussed.

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# ON THE BEHAVIOUR OF SOLUTIONS OF STOCHASTIC DIFFERENTIAL EQUATIONS

by

G. Kersting

*University of Frankfurt  
West Germany*

## ABSTRACT

In order to analyze the behaviour of some solution of the autonomous SDE

$$dX_t = b(X_t)dt + a(X_t)dW_t,$$

driven by a Wiener process  $(W_t)$ , one may try to compare it with solutions of the dynamical system  $dY_t = b(Y_t)dt$ . In the 1-dimensional situation the following turns out to be true: If  $a(x)^2$  is small compared to  $b(x)$ , then both systems show a similar behaviour. More precisely: If  $a(x)^2 = o(xb(x))$ , as  $x \rightarrow \infty$ , and if  $b(x) > 0$ , then  $X_t \sim Y_t$  for  $t \rightarrow \infty$  on the event  $\{X_t \rightarrow \infty\}$ . In the opposite situation  $xb(x) = o(a(x)^2)$  the facts are completely different. For example it is fairly easy to show that there is a monotone transformation  $f(x)$  with  $f(x) \sim x$  for large  $x$ , such that the drift term of the diffusion  $f(X_t)$  vanishes everywhere. Since this is not the case for  $f(Y_t)$  it is fairly obvious that  $X_t$  and  $Y_t$  are no longer close to each other.

We present a result, which generalizes this second assertion to non-degenerate diffusions in higher dimensions. No assumptions on the specific form of  $a(x)$  and  $b(x)$  are needed, also no knowledge on the qualitative behaviour of  $(X_t)$  or  $(Y_t)$  is required. The main assumption concerns the magnitude of the (the eigenvalues of)  $a(x)^2$  in comparison with  $|b(x)| \cdot |x|$  for large  $|x|$ .

Technically we are led to the problem of solving a certain Poisson equation  $Lf = g$  on the unbounded domain  $\{x \in \mathbb{R}^d : |x| \geq 1\}$ , subject to the boundary condition  $|f(x)| = o(|x|)$ , as  $|x| \rightarrow \infty$ .

# LEAVING AN INTERVAL IN LIMITED PLAYING TIME

by

David Heath\*

*Cornell University*

*Ithaca, NY*

and

Robert P. Kertz\*\*

*Georgia Institute of Technology*

*Atlanta, GA*

## ABSTRACT

A player starts at  $x$  in  $(-G, G)$  and attempts to leave the interval in a limited playing time. In the discrete time problem,  $G$  is a positive integer and the position is described by a random walk starting at integer  $x$ , with mean increments zero, and variance increment chosen by the player from  $[0, 1]$  at each integer playing time. In the continuous time problem, the player's position is described by an Ito diffusion process with infinitesimal mean parameter zero and infinitesimal diffusion parameter chosen by the player from  $[0, 1]$  at each time instant of play. To maximize the probability of leaving the interval  $(-G, G)$  in a limited playing time, the player should play boldly by always choosing largest possible variance increment in the discrete-time setting and largest possible diffusion parameter in the continuous-time setting, until the player leaves the interval. In the discrete time setting, this result affirms a conjecture of Spencer. In the continuous time setting, the value function of play is also identified.

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# RANDOM WALKS AND CONVOLUTION EQUIVALENT DISTRIBUTIONS

by

Claudia Klüppelberg

*Universität Mannheim  
West Germany*

## ABSTRACT

Consider an M/G/1 queue with arrival rate  $\lambda$  and service-time distribution  $F$  of mean  $\mu$ . If  $\rho := \lambda\mu$  is less than 1, then the stationary waiting-time distribution exists and is given by

$$W(x) = \sum_{n=0}^{\infty} (1-\rho)\rho^n F_I^{n*}(x),$$

where  $F_I(x) := \mu^{-1} \int_0^x F(y)dy$  is the integrated tail distribution of  $F$ . Under certain conditions on  $F_I$  the class  $S(\gamma)$  of convolution equivalent distributions is the characterizing class for tail-equivalence of  $W$  and  $F_I$ , where a distribution  $F$  belongs to  $S(\gamma)$ ,  $\gamma \geq 0$ , if

- (i)  $\lim_{x \rightarrow \infty} F(x-y)/F(x) = e^{\gamma y} \quad \forall y \in \mathbb{R}$
- (ii)  $\lim_{x \rightarrow \infty} F^{2*}(x)/F(x) = 2\hat{f}(\gamma) < \infty$

with moment generating function  $\hat{f}$  of  $F$ . Results of this type have been proved by Cline and Veraverbeke.

Thus one is interested in conditions on  $F$  implying  $F_I \in S(\gamma)$ . For a distribution  $F$  satisfying (i) Karamata's theorem gives  $\lim_{x \rightarrow \infty} F(x)/F_I(x) = \gamma$ . For  $\gamma > 0$  this ensures  $F_I \in S(\gamma)$  iff  $F \in S(\gamma)$ . In the case  $\gamma = 0$ ,  $F_I \in S(0)$  if  $F$  belongs to  $S^*$ .  $S^*$  is a subclass of  $S(0)$ , consisting of those distributions  $F$  satisfying

$$\lim_{x \rightarrow \infty} \int_0^x \{F(x-y)/F(x)\} F(y)dy = 2\mu < \infty.$$

Moreover, the class  $S^*$  leads to a characterization of  $S(\gamma)$  for  $\gamma > 0$  and thus provides the possibility to find uniform conditions for  $F \in S(\gamma)$  for any  $\gamma \geq 0$ .

If the function  $e^{\gamma x} F(x)$  is decreasing to 0 we prove

$$F \in S(\gamma) \iff G \in S^*$$

where  $G(x) := e^{\gamma x} F(x)$ . This result can be extended to characterize the class  $S(\gamma)$ .

# MULTICHANNEL QUEUEING SYSTEM WITH SEMI-ORDERED ENTRY

by

Masanori Kodama

*Kyushu University  
Fukuoka, Japan*

and

Jiro Fukuta

*Aichi University  
Toyohashi, Japan*

## ABSTRACT

We consider the multichannel queueing system with semi-ordered entry. The following assumptions are made for system operation:

- (1) For the system: System has  $m$  parallel servers and channels of each server are denoted by  $1, 2, \dots, m$  respectively.
- (2) For arrivals: (i) All arrivals arrive at a common entry for the system; (ii) arrivals are Poisson distributed, with mean arrival rate  $= \lambda$ .
- (3) For service: (i) Each server has a maximum waiting space capacity  $N_i$  for the server of the  $i$ th's channel ( $N_i \geq 1$ ); (ii) service time is exponentially distributed for each server, with mean service rate  $= \mu_i$  for the  $i$ th server; (iii) the time for service of a given server is independent of all prior service history, occupation of all servers and the size of waiting space.
- (4) For the queue discipline: (i) Arrival to the system checks each queue-length in front of channel, and he seeks the channels with the minimum length of queues unoccupied fully. And he enter to the one with minimum number in the channels sought above. We will call this as semi-ordered discipline. (ii) Arrivals are denied service if all waiting spaces are occupied.

In this paper, the stationary solutions about these systems are obtained and these results are compared with those ordered entry queue. The model described here has wide applications; for example, to production systems with closed-loop conveyors.

# A COMPUTATIONAL APPROACH TO NONLINEAR FILTERING BASED ON GAUSS INTEGRATION

by

Franz Konecny

*Universität für Bodenkultur, Wien  
Wien, Austria*

## ABSTRACT

Let  $X$  denote an one-dimensional diffusion process satisfying an Ito - differential equation

$$dX_t = f(X_t)dt + \sigma(X_t)dV_t.$$

It is required to estimate  $\phi(X_t)$  from an observation process of the form

$$dY_t = h(X_t)dt + \rho dW_t.$$

We address the problem of the recursive calculation of the conditional expectation  $E(\phi(X_t)|Y_s; s \leq t)$ . The corresponding conditional law  $\sigma_t$  can be represented in unnormalized form by the measure valued stochastic differential equation

$$d\sigma_t(\phi) = \sigma_t(L\phi)dt + \frac{1}{\rho^2}\sigma_t(h\phi)dY_t,$$

where  $L$  is the infinitesimal generator of  $X$ . By the classical Gaussian integration rules, we can write

$$\sigma_t(\phi) = \sum_{k=1}^n a_k(t)\phi(x_k(t)) + R_n.$$

The weights  $a_k(t)$  and Gauss-points  $x_k(t)$  are functionals of the observed path  $(Y_s; s \leq t)$  and the rest term  $R_n$  vanishes when  $\phi$  is a polynomial of order less than  $2n$ . The Galerkin equations for the processes  $(a_k(t), x_k(t); k < n)$  will be derived and analysed.

# WHITE NOISE APPROACH AND APPROXIMATIONS FOR TWO-PARAMETER FILTERS

by

H. Korezlioglu

*Ecole Nationale Supérieure des Télécommunications  
Paris, France*

## ABSTRACT

The filtering problem for processes with two continuous parameters was modelled and solved in (4) for the linear Gaussian case. The same model was used in (3) for the derivation of nonlinear filtering equations for two-parameter semimartingales; but no method of solution was proposed in the nonlinear case. The main difficulty in this case is due to the fact that the filtering problem can only be expressed and solved (if possible!) by means of filtering techniques for infinite dimensional processes. In the linear Gaussian case, the problem reduces to the resolution of Riccati equations for nuclear operator valued covariances.

We propose an approximation method of the filter by discretizing the parameters as in (2) and compare the method with the white noise approach, considered in (1). In case the state process is a bidirectional diffusion (1), the filter can be approximated by means of a recursively computable discrete parameter filter.

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# TRAVELLING WAVES IN BRANCHING DIFFUSION

by

S. P. Lalley

and

T. Sellke

*Purdue University*

## ABSTRACT

We study an inhomogeneous branching diffusion process in which individual particles execute standard Brownian motions and reproduce at rates depending on their locations. The rate of reproduction for a particle located at position  $x$  is  $\beta(x) = b + \beta_0(x)$ , where  $b > 0$  and  $\beta_0(x)$  is a nonnegative, continuous, integrable function. If  $M(t)$  is the position of the rightmost particle at time  $t$ , then as  $t \rightarrow \infty$ ,  $M(t) - \text{med}(M(t))$  converges in law to a location mixture of extreme value distributions. We determine  $\text{med}(M(t))$  to within a constant  $+o(1)$ . The linear rate at which  $\text{med}(M(t)) \rightarrow \infty$  depends on the largest eigenvalue of a differential operator involving  $\beta(x)$ ; when  $b > 0$ , the cases  $\lambda < 2$ ,  $\lambda = 2$ , and  $\lambda > 2$  are qualitatively different.

# LOW DENSITY EXPANSION FOR A TWO-STATE RANDOM WALK IN RANDOM ENVIRONMENT

by

Gregory F. Lawler

*Duke University  
Durham, NC*

## ABSTRACT

A nearest neighbor random walk on  $Z^2$  is considered where points of the lattice are labeled "good" or "bad". A particle takes a vertical step with probability  $a_G$  or  $a_B$  and a horizontal step with probability  $1 - a_G$  or  $1 - a_B$ , depending on whether its present site is good or bad. The increments are symmetric in the sense that a step of  $+1$  is as likely as a step of  $-1$ . If the good and bad sites are placed randomly, with density  $\rho$  of bad sites, it is known that there exists an  $a$  such that for almost every placement of sites, the random walk in the long run behaves like a homogeneous walk with vertical probability  $a$  and horizontal probability  $1 - a$ . Here we consider the problem of estimating  $a$  as a function of  $\rho$ ; in particular, when  $a_G = \frac{1}{2}$ ,  $a_B$  fixed, we derive rigorously the first two terms of the expansion of  $a(\rho)$  at  $\rho = 0$ .

# RKHS FOR GAUSSIAN MEASURES ON METRIC VECTOR SPACES

by

Anna T. Lawniczak

*University of Toronto  
Toronto, Canada*

## ABSTRACT

The RKHS for Gaussian measures on complete separable metric vector spaces which are not necessarily locally convex are constructed.

Let  $(X, B, \mu)$  denote a complete separable metric vector space with the Borel  $\sigma$ -algebra  $B$  completed in a symmetric Gaussian measure  $\mu$ . Let  $(R, B(R))$  denote the real line and  $B(R)$  its Borel  $\sigma$ -algebra. A measurable map  $F$  from  $(X, B)$  into  $(R, B(R))$  is called a quasi-additive measurable functional (q.m.f.) if

$$\begin{aligned} F(x+y) &= F(x) + F(y) & \mu \times \mu & \text{ a.e.} \\ F(-x) &= -F(x) & \mu & \text{ a.e.} \end{aligned}$$

Let  $X_\mu^*$  (the space of all q.m.f.) generate  $B(\text{mod } \mu)$  then the RKHS of the measure  $\mu$  coincides with the set of all admissible translates of the measure  $\mu$ .

Important examples of spaces of this kind which are not locally convex vector spaces are: the space  $L_0 = L_0[0, 1]$  of all measurable functions defined on the unit interval with convergence in measure or more generally some Orlicz spaces  $L_\Phi$ .

As an application of our result we obtain that every Gaussian measure defined on a separable, complete metric vector space such that there are sufficiently many measurable linear functionals is an extension of the canonical cylindrical Gaussian measure on the RKHS.

# SYSTEMS OF INDEPENDENT MARKOV CHAINS

by

T. M. Liggett

and

S. C. Port

*University of California, Los Angeles*

## ABSTRACT

Let  $P(x, y)$  be the transition probabilities for an irreducible Markov chain on a countable set  $S$ . We consider infinite systems of particles which move independently on  $S$ , following the law of this Markov chain. These systems are regarded as Markov processes on the space of configurations of particles on  $S$ . Since at least the time of Doob's classic book on stochastic processes, it has been known that one class of equilibrium distributions for this process consists of Poisson processes with intensities  $\pi$  which satisfy  $\pi P = \pi$ . Mixtures of these Poisson processes (which are called Cox processes) are also equilibrium distributions. In 1978, the first author proved that a sufficient condition for all equilibrium distributions for this system to be Cox processes is that

$$(C) \quad \lim_{n \rightarrow \infty} \sup_{x \in S} P^n(x, y) = 0$$

for all  $y \in S$ . While this is a natural condition in that it rules out positive recurrent chains, and corresponds to the infinitesimal condition for Poisson convergence of sums of independent indicators, examples suggest that one should determine the extent to which this condition is necessary for the conclusion to hold. We prove that it is not at all necessary if  $P$  is null recurrent, but that it is almost necessary if  $P$  is transient. (In fact, it is necessary if the chain satisfies the mild condition  $\sum_x P(x, y) < \infty$  for each  $y$ . When condition (C) fails, we construct non Cox equilibria by using entrance laws.) This state of affairs is particularly nice, since in many examples, it turns out that condition (C) either fails, or is hard to verify, if the chain is null recurrent. However, it is usually fairly easy to check whether or not it is satisfied if the chain is transient. In addition to studying the equilibria for these systems, we give conditions for convergence to a Cox equilibrium for general initial distributions.



# ERGODICITY AND INEQUALITIES FOR CERTAIN POINT PROCESSES

by

Torgny Lindvall

*University of Göteborg  
Göteborg, Sweden*

## ABSTRACT

Let  $N(A)$  be the number of renewals in the Borel set  $A \subset (0, \infty)$  for a zero-delayed renewal process, with lifelength distribution  $F$  having mean  $\mu$ . For simplicity, assume  $F$  has a density, and let  $\lambda = 1/\mu$  denote the renewal intensity. There are two categories of results on  $N$  and the renewal measure  $M$  ( $M(A) = E[N(A)]$ ); we may let them be represented by

- (1)  $\|M(t + \cdot) - \lambda \cdot \ell\|_c \rightarrow 0$  as  $t \rightarrow \infty$   
for all  $c > 0$ , where  $\|\cdot\|_c$  denotes total variation norm on  $[0, c]$  and  $\ell$  the Lebesgue measure,

and

- (2)  $N(t + A)$  is stochastically decreasing as  $t \rightarrow \infty$  for all Borel sets  $A$ , if  $F$  is of decreasing failure rate (DFR) type.

In this talk, we show that the analogue to (1) holds for a class of point processes with limited memory. We also introduce a way to define the DFR property for point processes in terms of partial orderings on the space of their histories. A suitable coupling, using imbedding in a bivariate Poisson process, is then instrumental in generalizations of (2).

# LATTICE BESSEL FUNCTIONS AND THEIR APPLICATIONS TO A TRANSIENT ANALYSIS OF QUEUEING NETWORKS

by

William A. Massey

*AT&T Bell Laboratories  
Murry Hill, NJ*

## ABSTRACT

Modified Bessel functions are the key to solving the joint transient distribution of the M/M/1 queue and the first time it becomes idle. To solve the analogous problem for the N-node series Jackson network, the author was led to invent a new class of special functions that generalize modified Bessel functions. They are called *lattice Bessel functions* since they are indexed by the N-dimensional integer lattice. A solution was obtained for the total busy period (every server is working) of a series Jackson network by analyzing the symmetry properties of these functions. We use the methods of images for the solution by showing that our state space is a fundamental domain of the lattice with respect to the group action induced by the lattice Bessel function symmetry group. Given a transient description of the series Jackson network before some server becomes idle, a natural follow-up issue is to solve for the joint distribution of the network precisely at the time that some queue empties. This was addressed in a subsequent paper (co-authored by Francois Baccelli of INRIA) using martingale techniques. Moreover, we can approximate these transient distributions by deriving their large time asymptotic expansions. This was achieved by computing the large time asymptotic expansions for lattice Bessel functions in a third paper, co-authored by Paul Wright of Berkeley. In a fourth paper co-authored by both F. Baccelli and P. Wright, we adapt these techniques to solving the analogous problem for a closed N-node cyclic network. Here we use an infinite symmetry group that decomposes into the semi-direct product of simpler groups. This allows us in turn to derive the spectral decomposition for the generator of this joint process. Finally, a fifth paper by the author applies these functions to derive the transient behavior of a series queueing network that is related but not equal to the Jackson network. The symmetry methods used for solving its transient distribution permit a derivation of its non-product form steady state measure.

# ASYMPTOTIC NORMALITY IN NONLINEAR FILTERING VIA A BAYESIAN CRAMÉR-RAO INEQUALITY

by

Eddy Mayer-Wolf

*Technion I.I.T.  
Haifa, Israel*

## ABSTRACT

It is well known that the problem of the nonlinear filtering of a diffusion is usually unsolvable in closed form. In recent years the diffusion's asymptotic lower order conditional moments have been identified, the small parameter being the intensity of the observation noise.

This work extends the asymptotic analysis by considering the diffusions' conditional law itself. Specifically, it is shown that under appropriate assumptions this conditional law, suitably rescaled, is asymptotically Gaussian.

The technique used involves the Fisher information  $J(\mu)$  of a probability measure  $\mu$  and a Bayesian Cramér-Rao type inequality, a scalar version of which is

$$(CR) \quad \text{var}(\mu) J(\mu) \geq 1$$

for which equality is achieved iff  $\mu$  is Gaussian. The idea is that asymptotic equality in (CR) yields asymptotic normality of  $\mu$ .

# A STOCHASTIC DIFFERENTIAL EQUATION INVOLVING CYLINDRICAL BROWNIAN MOTION

by

David McDonald

*University of Ottawa  
Ottawa, Canada*

## ABSTRACT

Let  $A$  be a constant, positive definite, self-adjoint operator on a real, separable Hilbert space  $H$  (in practice  $\ell^2$ ). We assume that  $A$  has a complete orthonormal family of eigenvectors  $\phi_k$  corresponding to a set of positive eigenvalues  $\lambda_k$ :

$$A\phi_k = \lambda_k \cdot \phi_k \quad k = 1, 2, \dots$$

We study the stationary, weakly continuous solution  $X(t)$  of the equation

$$dX(t) = -AX(t)dt + \sqrt{2a} dB(t)$$

where  $B(t)$  is a cylindrical Brownian motion on  $H$  and  $a$  is a constant, positive operator such that  $(\phi_k, \sqrt{a}\phi_k) = \sqrt{a_k}$ ;  $(\phi_i, \sqrt{a}\phi_j) = 0, i \neq j$ .

**Theorem.** Let  $T > 0$ . Assume that  $\sum_{k=1}^{\infty} \frac{a_k}{\lambda_k} < \infty, \sum_{k=1}^{\infty} \frac{a_k^2}{\lambda_k} < \infty$  and the ratios  $(a_k/\lambda_k)_{k=1}^{\infty}$  are distinct (with  $a_1/\lambda_1 = \max_k (a_k/\lambda_k)$ , without loss of generality). Then

$$\begin{aligned} &P\left(\sup_{t \in [0,1]} \|X(t)\| > x\right) \\ &\leq \left[2e\lambda_1^{3/2}a_1K_1x \exp\left(-\frac{\lambda_1}{2a_1}x^2\right)\right] \cdot \left[T(1 + O(x^{-2})) + \frac{a_1}{2}\lambda_1^{-2}x^{-1} + O(x^{-2})\right] \end{aligned}$$

where  $K_1 = \left(\frac{1}{a_1}\right)^{3/2} \cdot \frac{1}{\sqrt{2\pi}} \cdot \prod_{k=2}^{\infty} \left(1 - \frac{a_k\lambda_1}{\lambda_k a_1}\right)^{-1/2}$ , and the two  $O$ -terms are with respect to  $x \rightarrow +\infty$ , and are uniform in  $T > 0$ .

These results are also extended to linear feedback control systems driven by Gaussian noise and one calculates the probability of an large deviation of a quadratic cost function of the state  $X(t)$ .

# STOCHASTIC FLOW NETWORKS

by

Avi Mandelbaum

*Stanford University  
Stanford, CA*

## ABSTRACT

This talk, based on a joint work with Hong Chen, is about a stochastic model for flow in a network. The network is represented by a graph, and the flow is that of indistinguishable units which traverse individually along its arcs and interact within its nodes. Two familiar examples where such a model arises are some queueing and finite particle systems. Three levels of space-time aggregation are discussed. First, the model is introduced at a microscopic level, paying attention to its fine structure. Then, macroscopic analysis identifies the heavily congested nodes. Finally, a diffusion approximation, in terms of the heavily congested nodes, is proposed at an intermediate level of aggregation. When the number of units that circulate within the network is constant in time, this diffusion approximates the number of units present at each node (as a fraction of total), and it behaves like a Brownian motion that is reflected off the faces of the unit simplex.

# THE TRANSIENT BEHAVIOR IN ERLANG'S MODEL FOR LARGE TRUNK GROUPS AND VARIOUS TRAFFIC CONDITIONS

by

Debasis Mitra

and

Alan Weiss

*AT&T Bell Laboratories  
Murray Hill, NJ*

## ABSTRACT

We consider the classic queuing problem in telephony: obtaining the probability that a customer who attempts to use a phone finds no circuit available. Specifically, we suppose that a group of  $N$  trunks (circuits) is empty at time  $t = 0$ , and thereafter, receives a Poisson ( $\lambda$ ) stream of calls, each call having an independent, exponentially distributed holding time with mean one (1). This is an M/M/N/N queue. We let  $E(t; N, \lambda)$  denote the probability of blocking; that is,  $E(t; N, \lambda)$  is the probability that all  $N$  circuits are in use at time  $t$ . By scaling  $\lambda$  with  $N$  so that  $\lambda = N\gamma$ , we obtain asymptotic expressions for  $E(t; N, N\gamma)$  as  $N$  becomes large for three natural cases:  $\gamma < 1$  (light traffic),  $\gamma = 1$  (moderate traffic), and  $\gamma > 1$  (heavy traffic).

We also obtain sample path results. For example, let  $Z_N(t)$  denote the fraction of trunks in use at time  $t$  ( $0 \leq Z_N(t) \leq 1$ ), and define  $W_T(t) = \gamma(1 - e^{-t}) + C(e^t - 1)$ , where  $C = \frac{1-\gamma(1-e^{-T})}{\gamma-1}$ . Conditioned on  $Z_N(T) = 1$ , we can show that for any  $\epsilon > 0$ ,

$$\lim_{N \rightarrow \infty} P\left(\sup_{0 \leq t \leq T} |Z_N(t) - W_T(t) + \epsilon Z_N(t) - 1| > \epsilon\right) = 0.$$

(If  $\gamma > 1$  we need  $T < \log \frac{\gamma}{\gamma-1}$  for this result to hold.)

The results are derived in two ways. Most of the asymptotic expansions come from the Laplace transform of  $E(t; N, \lambda)$ , which we expand using Laplace integral asymptotics and then invert. The sample path results are obtained from the theory of large deviations.

# STOCHASTIC MODELS OF QUEUE STORAGE

by

E. G. Coffman, Jr., L. Flato, I. Mitrani, L. A. Shepp

*AT&T Bell Laboratories  
Murray Hill, NJ*

and

C. Knessl

*Northwestern University  
Evanston, IL*

## ABSTRACT

We study three queueing models where storage cells are occupied by incoming items and are released by departing ones. Under those conditions, storage tends to fragment. A free cell may have a lower address than an occupied one, in which case the former is said to be 'wasted'. The object of interest is the equilibrium distribution of the stochastic process  $[X(t), Y(t)]$ , where  $X(t)$  is the number of wasted cells and  $Y(t)$  is the number of items present, at time  $t$ .

In all three cases, items arrive according to a Poisson process and have exponentially distributed required service times. In the first model, there are infinitely many servers and the storage scheduling strategy is 'First-Fit'. A closed form solution for the number of wasted cells is obtained. In the second model, there is a single server and the scheduling strategy is 'Leftmost Preemptive-Resume'. The interest of this model is that it provides an upper bound on the number of wasted cells, for a large class of storage strategies. A series expansion is obtained for the generating function of the joint distribution of  $X$  and  $Y$ . This yields a simple expression for the asymptotic behaviour of  $E(X)$  under heavy load.

The third model is also a single-server one, with a Processor-Sharing service discipline and a 'Periodic Compaction' storage strategy. It bounds the number of wasted cells from below, at least within the class of Processor-Sharing strategies. The problem of finding the joint distribution of  $X$  and  $Y$  is reduced to that of solving a set of one-dimensional recurrence relations. Again, there is a simple asymptotic expression for  $E(X)$  under heavy load.

**HOMOGENEOUS RANDOM MEASURES FOR MARKOV  
PROCESSES IN WEAK DUALITY: STUDY VIA AN  
ENTRANCE BOUNDARY**

by

H. Kaspí

*Technion  
Haifa, Israel*

and

J. B. Mitro

*University of Cincinnati  
Cincinnati, OH*

**ABSTRACT**

For Markov processes in weak duality, we study time changes, decompositions of Revuz measure, and potentials of additive functionals which may charge  $\zeta$ , the process lifetime. The basic tools are a Ray-Knight (entrance) compactification, Dynkin's theory of minimal excessive measures, and a process with random birth and death. The talk will focus on an application of these results to entrance laws for one-dimensional diffusions on  $(0, \infty)$ .



# REMARKS ON THE BASIC EQUATIONS FOR A SUPPLEMENTED GSMP AND ITS APPLICATIONS TO QUEUES

by

Masakiyo Miyazawa

*Science University of Tokyo*

and

Genji Yamazaki

*Tokyo Metropolitan Institute of Technology*

## ABSTRACT

A supplemented GSMP (Generalized Semi-Markov Process) is known as a useful stochastic process for discussing fairly general queues including queueing networks. Although much work has been done on its insensitivity, there were only a few works on its general discussion. This paper considers a supplemented GSMP in general setting. Our main concern is a system of Laplace-Stieltjes transforms of the steady state equations called the basic equations. The difference between the basic equations and the ordinary ones is that the former use Palm distributions of point processes. We first derive the basic equations under the stationary condition based on theory of point processes. It is shown that those basic equations with some additional conditions characterize a stationary distribution of GSMP. That is, they give a generator of a supplemented GSMP. We also discuss how to get the stationary distribution when a solution of the basic equations is partially known or inferred. This includes an important remark to the fact by which we are liable to be trapped. Some examples of queues are given which includes a counter example to the literature.

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# DECOMPOSITION OF BINARY RANDOM FIELDS, AND SOME RELATED TOPICS FROM STATISTICAL MECHANICS

by

Charles M. Newman

*University of Arizona  
Tucson, AZ*

## ABSTRACT

Let  $\delta_c(X)$  denote the maximum  $d$  in  $[0, 1/2]$  such that a binary random field  $X$  can be decomposed as the modulo 2 sum of two independent binary fields, one of which is independent Bernoulli (white binary noise) of weight  $d$ . This quantity is of some relevance in the analysis of the information theoretic rate distortion function of  $X$ . Lower bounds on  $\delta_c$  when  $X$  is Gibbsian have been obtained by Bassalygo-Dobrushin (*Problems Info. Trans.* (Russian), **23** no. 1 (1987)), Hajek-Berger (*Ann. Prob.*, **15** no. 3 (1987)) and the author (*Ann. Prob.*, **15** no. 3 (1987)). Our methods are based on the location of the complex zeros of the probability generating function of  $X$ . We discuss these methods and point out that in certain cases they lead to an exact relation between  $\delta_c$  and a well known (and numerically computable) statistical mechanical quantity, the radius of convergence of the Mayer expansion.

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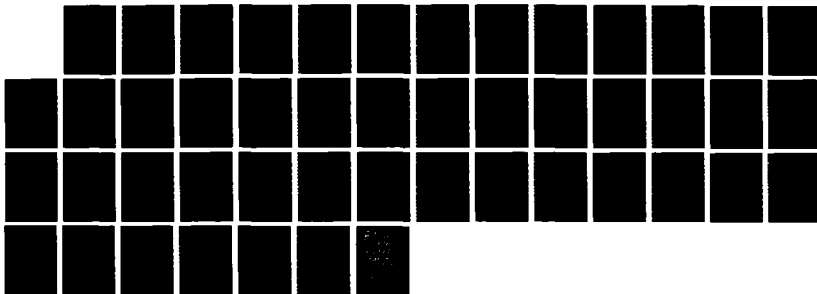
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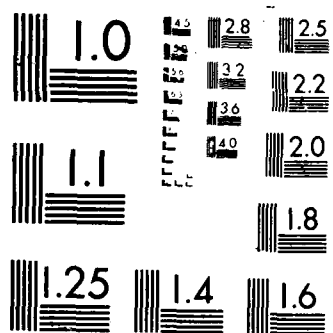
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# ON THE LEVY-PROHOROV DISTANCE BETWEEN COUNTING PROCESSES

by

Martti Nikunen

and

Esko Valkeila

*University of Helsinki  
Helsinki, Finland*

## ABSTRACT

Suppose that  $N$  is a counting process having a deterministic continuous compensator  $A$  and let  $M$  be another counting process on the same probability space with compensator  $B$ . In [3] we derived an upper bound for the Levy-Prohorov distance between the distributions of  $N$  and  $M$  in the case where  $B$  is continuous. Now we extend this result to some cases in which the compensator  $B$  can have jumps. This result gives some information about the rate of convergence in a limit theorem due to Kabanov, Liptser and Shiriyayev [1].

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AN EXISTENCE THEOREM FOR MEASURES ON  
PARTIALLY ORDERED SETS, WITH APPLICATIONS  
TO RANDOM SET THEORY

by

Tommy Norberg

*University of Göteborg and  
Chalmers University of Technology  
Göteborg, Sweden*

ABSTRACT

We state conditions on a partially ordered set  $L$  and a mapping  $\lambda$  defined on a class  $\mathcal{F}_C$  of filters on  $L$ , under which the latter extends to a measure on the minimal  $\sigma$ -field over  $\mathcal{F}_C$ . By applying this extension result to the case when  $L$  is a continuous lattice with a second countable Scott topology, we obtain a characterization of the probability measures on  $L$ . The correspondence on the line between probability measures and distribution functions is a special case of this characterization. Another special case is a variant of Choquet's existence theorem for distributions of random closed sets in locally compact second countable Hausdorff spaces  $S$ . Our approach to this result shows that it holds as soon as the topology of  $S$  is continuous and second countable. We also obtain characterizations of the distributions of all random compact and all random compact convex subsets in  $R^d$  for finite  $d$ .

# CONSTRAINED DYNKIN'S STOPPING PROBLEM WITH CONTINUOUS PARAMETER

by  
Yoshio Ohtsubo  
*Kochi University*  
*Japan*

## ABSTRACT

In this paper we study Dynkin's stopping problem with a finite constraint for right continuous, adapted and bounded processes, which is an extension of a discrete time case [2]. In general a minimax value process of the problem does not coincide with a maximin process. We show that there exists a right continuous version of the minimax (maximin) value process which has the aggregation property, and that the version is the smallest (largest) among right continuous processes satisfying certain stopped super- and submartingale inequalities. We also give the necessary and sufficient conditions for the coincidence of two value processes. Under these conditions the value process is a unique solution to martingale inequalities.

For Dynkin's problem without a finite constraint Lepeltier and Maingueneau [1] and Stattner [3] have already proved the aggregation and the coincidence of the value processes without any conditions.

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# WHEN IS A STOCHASTIC INTEGRAL A TIME CHANGE OF A DIFFUSION?

by

Bernt Øksendal

## ABSTRACT

We give a necessary and sufficient condition (in terms of  $u, v, b, \sigma$ ) that a time change of an  $n$ -dimensional Ito stochastic integral  $X_t$  of the form

$$dX_t = u(t, \omega)dt + v(t, \omega)dB_t$$

leads to a process with the same law as a diffusion  $Y_t$  of the form

$$dY_t = b(Y_t)dt + \sigma(Y_t)dB_t,$$

where the generator  $A$  of  $Y_t$  is assumed to have a unique solution of the martingale problem. The result has applications to filtering theory, conformal martingales in  $\mathcal{C}^\infty$  and harmonic morphisms.



ON THE IBRAGIMOV-IOSIFESCU CONJECTURE FOR  
 $\phi$ -MIXING SEQUENCES

by

Magda Peligrad

*University of Cincinnati  
Cincinnati, OH*

ABSTRACT

The aim of this paper is to give new central limit theorems and invariance principles for sequences of  $\phi$ -mixing sequences of random variables that come to support the truth of the Ibragimov-Iosifescu conjecture. A related conjecture is formulated and a positive answer is given for the distributions that have tails regularly varying with the exponent  $-2$ .

# **STRONG APPROXIMATION OF RECORDS AND RECORD TIMES BY POISSON AND WIENER PROCESSES**

by

**Dietmor Pfeiffer**

## **ABSTRACT**

For an i.i.d. sequence of random variables we investigate the asymptotic strong behavior of record values and record times. Several approaches are considered which relate record times with Poisson processes, which gives rise to a strong approximation by Wiener processes in the sense of Komlos-Major-Tusnady. Interesting new relationships between record times and the jump times of extremal processes as well as the record values are among the results to be presented.

# OPTIMAL INSPECTION AND CONTROL OF A SEMIMARKOV DETERIORATION PROCESS

by

Armando Z. Milioni

*Northwestern University  
Evanston, IL*

and

Stanley R. Pliska

*University of Illinois  
Chicago, IL*

## ABSTRACT

Systems operating while undergoing deterioration toward failure are common situations. When modeling such systems it is customary to model that underlying deterioration process, be it the wear of a cutting tool, the length of microcracks in a mechanical part, or the size of a cancerous tumor, as a Markov process, even though such a submodel is often unrealistic. As a first step toward an accurate model, we take the underlying deterioration process to be semimarkov.

The process can be in one of three states, say healthy, defective, and sick. We seek the inspection schedule which minimizes the expected costs of inspection, treatment, and sickness. False positives can occur, but when they do an additional, more expensive test reveals the true state. The problem is solved with (what else?) dynamic programming. The value function, which is a function only of the process' age, is right-continuous but not necessarily decreasing.

Our basic model is extended to the cases where costs are discounted and where false positives cannot be distinguished from true positives. We also give particular attention to the catastrophic case where the cost of sickness is infinite. Two versions are analysed: the problem of minimizing the probability of sickness subject to a specified upper bound on the number of inspections, and the problem of minimizing the expected inspection and treatment costs subject to a specified upper bound on the probability of sickness.

# SOME LIMIT THEOREMS FOR DIFFUSION PROCESSES ON THE CIRCLE AND THE ANNULUS

by  
Ross Pinsky

## ABSTRACT

Let  $L_\epsilon = \epsilon L_0 + L_1$ , where  $L_0$  is a nondegenerate elliptic operator on  $R^2$  and  $L_1 = \frac{1}{2}A(r, \theta)\frac{\partial^2}{\partial \theta^2} + B(r, \theta)\frac{\partial}{\partial \theta}$ . We assume that for fixed  $r$ ,  $L_1$  generates a positive recurrent diffusion on the circle with invariant measure  $\mu_r(d\theta)$ . Let  $X^\epsilon(t) = (r^\epsilon(t), \theta^\epsilon(t))$  denote the diffusion generated by  $\frac{1}{\epsilon}L_\epsilon$  and let  $u_\epsilon$  be the solution to the Dirichlet problem  $L_\epsilon u = 0$  on  $D$  and  $u = f$  on  $\partial D$ , where  $D = \{x : r_1 < |x| < r_2\}$  and  $f$  is continuous. Then  $u_\epsilon(x) = E_x f(X^\epsilon(X^\epsilon(\tau_D^\epsilon)))$  where  $\tau_D^\epsilon$  is the first exit time from  $D$ . By the averaging principle, the process  $r^\epsilon(t)$  converges weakly to the process  $r^0(t)$  generated by  $L_0$ , the operator obtained from  $L_0$  by restricting to functions depending only on  $r$  and averaging the coefficients with respect to  $\mu_r(d\theta)$ . Furthermore,  $P_x(\theta^\epsilon(\tau_D^\epsilon)\epsilon d\theta | \tau_{r_i}^\epsilon < \tau_{r_j}^\epsilon)$  converges weakly to  $\mu_{r_i}(d\theta)$  as  $\epsilon \rightarrow 0$ , where  $\tau_r^\epsilon$  denotes the hitting time of the circle of radius  $r$  and  $(i, j) = (1, 2)$  or  $(2, 1)$ . The above information allows us to evaluate the limiting distribution of  $(r^\epsilon(\tau_D^\epsilon), \theta^\epsilon(\tau_D^\epsilon))$  and thus also the asymptotics of  $u_\epsilon(x)$ .

Now consider the situation in which we perturb the annulus  $D$ . Now, starting from  $x$  satisfying  $r_1 < |x| < r_0$ , one expects intuitively that the support of the exit distribution will converge to the limiting support consisting of the  $r_1$ -ball with the two points  $(r_0, \theta_i)$ ,  $i = 1, 2$  adjoined. What is not clear is how the limiting exit distribution is distributed on  $(r_0, \theta_1)$  and  $(r_0, \theta_2)$ . We will evaluate this limiting distribution which will then also allow us to evaluate the asymptotics of  $u_\epsilon(x)$  as  $\epsilon \rightarrow 0$ .

# ON THE GI/GI/1/0 QUEUEING LOSS SYSTEM

by

**Behnam Pourbabai**

*New York University  
New York, NY*

## ABSTRACT

In this paper both the overflow and the departure processes from a GI/GI/1/0 queueing loss system with a renewal input, independent and identically distributed service times, a single server, and no waiting room will be characterized as special regenerative processes. Furthermore, the tandem behavior of a GI/GI/1/0  $\rightarrow$  • /GI/1/0 queueing loss system will be studied by approximating the departure process from each queueing loss system with a compatible phase type renewal process. In addition, numerical results will be provided and the approximation outcomes will be compared against those from a simulation study. Finally, engineering applications will be discussed.

# ON THE DEPENDENCE STRUCTURE OF HITTING TIMES OF MULTIVARIATE PROCESSES

by

Nader Ebrahimi

and

T. Ramalingam

*Northern Illinois University  
DeKalb, IL*

## ABSTRACT

The subject of dependence concepts and related inequalities for random variables and, more generally, for stochastic processes is one of the most widely studied topics in probability and statistics. Very often, analysis of dependent multivariate processes relies heavily on the study of the distributional aspects of two or more hitting or first-passage times. The properties of such random times have been well documented for processes exhibiting specific dependence structures such as the Markovian. Whereas such traditional settings have been fruitful, stochastic modeling of hitting times per se in order to account for positive or negative dependence among such times is often either necessary or expedient. For example, in many reliability problems, it may be possible to observe not the processes themselves but only terminal events such as failure or death of systems and any information as to the nature of dependence among first failure times will be useful in the estimation and prediction of the system reliability. Besides, such direct modeling of random times may bring forth new inequalities for the underlying processes.

Ebrahimi [*J. Applied Prob.*, 24 (1987)] has initiated a direct approach to the study of dependence structures of hitting times of bivariate processes. Following his approach, positive and negative dependence notions and properties thereof will be presented for more general multivariate processes. Inequalities for the zeros of the Brownian Motion process and for the reliability of systems will be given as applications of our concepts and results.

# **LIGHT TRAFFIC DERIVATIVES VIA LIKELIHOOD RATIOS**

by

**Martin I. Reiman**

and

**Alan Weiss**

*AT&T Bell Laboratories*

*Murray Hill, NJ*

## **ABSTRACT**

We consider the steady-state behavior of open queueing systems with Poisson arrival processes in light traffic, that is, as the arrival rate tends to zero. We provide expressions for derivatives of various quantities of interest (such as moments of steady state sojourn times and queue lengths) with respect to the arrival rate, at an arrival rate of zero. These expressions are obtained using the regenerative structure of the queueing system, along with a change of measure formula based on likelihood ratios. The derivatives, which can be used in interpolation approximations, may be evaluated analytically in simple cases, and by simulation in general.

# **SOME PROPERTIES OF WESTCOTT'S FUNCTIONAL**

by  
**Paul Ressel**

## **ABSTRACT**

For random measures on locally compact spaces the so-called Laplace functional is the appropriate generalization of the classical Laplace transform. These functionals may be characterized by positive definiteness and a weak continuity property. A certain sharper version of positive definiteness will be shown to single out the Westcott's functionals, i.e. the Laplace functionals of joint processes. A stronger continuity requirement characterizes finitary point processes.



# **SOME STOCHASTIC CONTROL PROBLEMS AND THEIR APPLICATIONS TO INEQUALITIES FOR DIFFUSIONS**

by

**Gareth Roberts**

*Warwick University  
United Kingdom*

## **ABSTRACT**

The problem of controlling a Brownian motion with a process of bounded velocity, so as to minimize certain functionals of its path, is considered. In particular the functionals,  $E[f(x_t)]$ ,  $E[\int_0^\infty e^{-\alpha s} f(x_s) ds]$ , and  $E[f \int_0^T I[x_s > t]]$ , for symmetric, increasing for  $x \geq 0$  functions  $f$ , are minimized. Stochastic bounds for a class of diffusions are thus founded, and this result is generalized to  $n$ -dimensional processes. Similarly, stochastic bounds for  $T_t$ , the time  $|x_t|$  spends above a certain level are found for the same class of diffusions. Explicit solutions are given for the first two problems.

# QUEUES WITH NON-STATIONARY INPUT STREAM

by

Tomasz Rolski

Wrocław University  
Poland

## ABSTRACT

We consider a single server system with general independent inter-arrival times and doubly stochastic Poisson arrivals. In this talk I shall survey the following issues:

- (i) ergodic properties for such queues and their stationary representations ([5]),
- (ii) bounds for the characteristics ([2] and references therein, [3], [6]),
- (iii) periodic Poisson queues as a special case of interest with emphasis on
  - inequalities and identities between queueing characteristics ([1]),
  - evaluation of characteristics.

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# A SPOT WELDING RELIABILITY PROBLEM

by

M. Rumsewicz

and

P. Taylor

*University of Adelaide  
South Australia*

## ABSTRACT

Many production lines have a station where spot welding takes place. At such stations it is frequently the case that multiple operators draw on the same power supply. Given the high voltage drops that occur during spot welding and the deterioration in weld quality that can occur when a large number of operators spot weld simultaneously, it is of interest to investigate the probability of poor welds being produced. We analyse a spot welding station used at General Motors-Holden (Aust). The first model reduces to a generalized Engset system (as used in telecommunications models). The second model removes access to the power supply of additional welders whenever the extra power drain would cause poor quality welds. We make the assumption that all relevant distributions are negative exponential so as to allow analytic tractability of the problem. The model then becomes one of the Quasi Birth and Death type as analysed by Neuts (1981) and Gaver et al (1984). Performance measures and numerical results are presented for the range of parameter values typical of those at GMH. These show the size of power supply required to adequately service a spot welding station is very small for either model. Hence it is preferable to implement the second model examined as this would lead to no poor quality welds being produced.

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# INSENSITIVITY AND GENERALISED TRANSITION RATES

by

M. Rumsewicz

*University of Adelaide  
South Australia*

## ABSTRACT

We consider a process  $P$  on a set of states  $S$ , and  $A$  is a subset of  $S$ . Utilizing Whittle's concept of insensitivity, we suppose that upon jumping into the set  $A$  the process is assigned a nominal sojourn time from a general distribution with unit mean. This system is said to be insensitive to nominal sojourn time in  $A$  if the equilibrium distribution of time spent in each state is unaffected by the form of the general distribution. Most of the current literature makes the assumption that all transition rates  $q(x, x')$  from state  $x$  to state  $x'$  are fixed. In this paper we relax this assumption and allow the transition rates to depend upon the length of time the process has been in  $A$  and show that under certain conditions the insensitivity of the process is maintained.

# ON A CLASS OF CUMULATIVE PROCESSES IN WARRANTY ANALYSIS AND PENSION ACCUMULATION

by

Izzet Sahin

*University of Wisconsin  
Milwaukee, WI*

## ABSTRACT

Consider a point process with time-varying interoccurrence intervals. For  $i = 1, 2, \dots$ , let the  $i$ th interval start at  $t_{i-1}$  and terminate at  $t_i$  where  $0 \equiv t_0 < t_1 < \dots$ . Write  $X(t_{i-1}) = t_i - t_{i-1}$ . Assume that the distribution of  $X(r)$  depends continuously on  $r$  but, given  $t_i$ ,  $X(t_i)$  does not depend on  $X(t_j)$ ,  $j < i$ . Let  $N(t)$  be the number of occurrences during  $(0, t]$ , and, for  $s \geq 0$  arbitrary but fixed, let  $U(s, r, X(r))$  represent a function of the interval  $X(r)$ , its starting epoch  $r$ , and the constant  $s$ . Define  $W(s, t) = \sum_{i=0}^{N(t)-1} U(s, t_i, X(t_i))$ ,  $N(t) > 0$  ( $W(s, t) = 0$  if  $N(t) = 0$ ) as the cumulative "reward" during  $(0, t]$ .

Time-dependent behavior of processes of the type  $\{W(s, t), t \geq 0\}$  and related questions are investigated. Some special cases of interest are: 1)  $U = 1$  if  $X \geq s$ ,  $U = 0$ , otherwise; in this case,  $W(s, t)$  is the number of occurrences during  $(0, t]$  that are preceded by intervals of length  $\geq s$ ; 2)  $U = X$  if  $X \geq s$ ,  $U = 0$ , otherwise; in this case,  $W(s, t)$  is the total time spent during  $(0, t]$  in sojourns of length  $\geq s$ ; 3)  $U = \min(s, X)$ , when  $W(s, t)$  represents the total time measured during  $(0, t]$ , if the clock is stopped whenever  $X > s$ . These processes have applications in warranty analysis and pension accumulation.

# STOCHASTIC EVOLUTION EQUATIONS WITH EXPONENTIAL TYPE FLUCTUATION

by

Takeyuki Hida

*Nagoya University  
Japan*

and

Kimiaki Saito

*Meijo University  
Japan*

## ABSTRACT

Exponential type fluctuations of the forms  $F(t) = \exp[\int f(t, u) \dot{B}(u) du]$  and  $G(t) = \exp[\int g(u) \dot{B}(u)^2 du]$ ,  $\dot{B}(t)$  being the Gaussian white noise, often arise in the stochastic evolution equations describing biological or physical phenomena. An important example of such a process is the so-called Oosawa's equation:

$$\frac{d}{dt} X(t) = -k_- F(t) \cdot X(t) + k_+ F(t)^{-1} \cdot (1 - X(t)),$$

where  $\int f(t, u) \dot{B}(u) du$  is taken to be the Ornstein-Uhlenbeck process with a positive constant. Using the white noise analysis we can discuss some evolutionary phenomena determined by the equation.

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# MAXIMA OF SYMMETRIC STABLE PROCESSES

by

Gennady Samorodnitsky<sup>1,2</sup>

*University of North Carolina  
Chapel Hill, NC*

## ABSTRACT

We study large deviations of symmetric stable processes represented as stochastic integrals with respect to random symmetric stable measures.

The exact asymptotic behavior of supremum distribution functions is established for symmetric stable processes with  $0 < \alpha < 1$ , as well as for those defined on finite parameter sets. For symmetric stable processes with  $1 \leq \alpha < 2$  defined on infinite sets, an asymptotic lower bound for the supremum distribution function is given.

We deduce a necessary and sufficient condition for a.s. boundedness of symmetric stable processes with  $0 < \alpha < 1$ , and a necessary condition for a.s. boundedness of symmetric stable processes with  $1 \leq \alpha < 2$ .

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<sup>1,2</sup> This research was conducted while the author was receiving Dr. Chaim Weizmann Post-Doctoral Fellowship for Scientific Research.

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# A MULTIPARAMETER ALMOST SUPERADDITIVE LIMIT THEOREM AND ITS APPLICATION TO COMBINATORIAL OPTIMIZATION

by

Klaus Schürger

*University of Bonn  
FR Germany*

## ABSTRACT

Motivated by the properties of a family of random variables which arises in the context of the traveling-salesman problem, we introduce a strong almost superadditivity condition (SASA) for multiparameter families of random variables which is weaker than a certain notion of superadditivity introduced in [1]. We derive an almost sure limit theorem for families of random variables satisfying the SASA, which generalizes an ergodic theorem of [1]. We also indicate how our limit theorem entails results on the growth of matchings, Steiner trees, traveling-salesman processes as well as triangulations in large areas. These applications have been motivated by [2], [3], [4].

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# THE DISTRIBUTION OF THE SPLITTING TIME FOR A DNA STRAND

by

L. A. Shepp

and

R. Vanderbei

*AT&T Bell Laboratories  
Murray Hill, NJ*

## ABSTRACT

Mark Westcott asked for the distribution of the time  $\tau$  for a strand of length  $L$  to completely unravel if an end of a strand unravels at rate  $1/2$ , and new end points of unraveling occur at rate  $\lambda dz dt$  along any remaining interval  $dz$  of the strand in any time interval  $dt$ . A direct approach involves higher dimensional Markov processes, but we show that the process can be reduced to a solvable one dimensional problem by introducing the new Markov process  $X = X(t)$ ,  $t \geq 0$  which increases at rate 1, and jumps at rate  $\lambda X(t)$  to a uniformly distributed random fraction of itself at each time  $t$ . The maximum of the  $X$  process starting at  $X(0) = 0$ , over an interval of length  $L$ , has the same distribution as  $\tau$ . We show by this method that  $\tau$  is essentially constant.

# EQUILIBRIUM IN A MULTI-AGENT CONSUMPTION/INVESTMENT PROBLEM

by

S. E. Shreve

and

J. P. Lehoczky

*Carnegie Mellon University  
Pittsburgh, PA*

## ABSTRACT

Each of finitely many agents is endowed with a commodity stream, and the agents must collectively, dynamically determine the price of this commodity for purposes of trading. The agents invest their income from trading in a market whose stock prices are continuous semimartingales. The agents may also consume the commodity, and do so in order to maximize the expected utility of consumption, subject to the condition of almost sure nonnegative wealth at the final-time. Equilibrium obtains when a commodity price process can be found so that each agent solving his own stochastic control problem results in aggregate consumption exactly equal to aggregate endowment, for all times, almost surely. Such an equilibrium will be exhibited.

# QUEUES AS HARRIS RECURRENT MARKOV CHAINS

by

Karl Sigman

*Columbia University  
New York, NY*

## ABSTRACT

We present a general framework for modeling queues in discrete time (at arrival epochs) as Harris recurrent Markov Chains (HRMC's). Since HRMC's are regenerative, the queues inherit the regenerative structure. The input to the queues is a marked point process (mpp) for which the sequence  $(T_n, k_n)$  of interarrival times and marks is assumed governed by a HRMC. Such inputs include the cases of i.i.d., regenerative, Markov modulated as well as the output from some queues. As specific examples, we consider split and match, tandem, G/G/c as well as more general open networks. In the case of i.i.d. input, explicit regeneration points can sometimes be found and we include some examples of this type.

*Keywords and phrases:* Queue, discrete time, Harris recurrent Markov chain, regenerative.

# ANATOMY OF THE INTERPOLATION METHOD FOR APPROXIMATING FUNCTIONS OF QUEUEING SYSTEMS

by

Burton Simon

*University of Colorado  
Denver, CO*

## ABSTRACT

Suppose  $\lambda$  is the total arrival rate of customers to a queueing system, and  $f(\lambda), 0 \leq \lambda < c$  is some function of interest such as a moment or quantile of a sojourn time distribution. The light traffic limits of  $f$  are its derivatives at  $\lambda = 0; f(0), f'(0), f''(0), \dots$ . A heavy traffic limit of  $f$  is  $h = \lim_{\lambda \rightarrow c} g(\lambda)$ , where  $g(\lambda)$  is a "normalized" version of  $f(\lambda)$ , e.g.  $g(\lambda) = (c - \lambda)f(\lambda)$ . Light and heavy traffic limits can be computed for a large class of systems, many of which are otherwise intractable. If  $k$  light traffic limits along with a heavy traffic limit can be calculated for  $f$  then  $f(\lambda)$  can be approximated by  $\hat{f}(\lambda) = \hat{g}(\lambda)/d(\lambda)$ , where  $\hat{g}(\lambda)$  is the unique  $k$ th degree polynomial that matches the known light and heavy traffic limits of  $g(\lambda)$ , and  $d(\lambda)$  is the normalizing function, i.e.  $g(\lambda) = d(\lambda)f(\lambda)$ .

This talk will present an overview of the interpolation method, and a new (simple but surprising) result that relates light and heavy traffic limits. This relation sheds light on the interpolation method by offering another construction of the same approximation, and points towards possible methods for attacking some open issues.

# RECURSIVE ESTIMATION FOR A CLASS OF COUNTING PROCESSES

by

Peter Spreij

*Free University  
Amsterdam, Netherlands*

## ABSTRACT

In this paper we will consider recursive parameter estimation for counting processes that admit an intensity process  $f$  which is linear in the parameter, meaning that  $f = x.p$ . Here  $x$  is a  $d$ -dimensional predictable process and  $p$  is a  $d$ -dimensional parameter. The dot means inner product. The algorithm that we want to present is similar to the one that can be found in [1]. The formula for the algorithm that we will give is derived from an asymptotic expression for the likelihood ratio process under the assumption that local asymptotic normality holds. In [1] we proved strong consistency under the restricting condition that the eigenvalues of a regressing type matrix tend to infinity with the same convergence rate for all eigenvalues. In this paper we considerably relax this condition in that it is now assumed that the minimal eigenvalue of this matrix tends to infinity and that the logarithm of the maximal eigenvalue is bounded by the minimal eigenvalue. A condition of this type is known for least squares estimation in linear regression models; see for instance [2,3]. Contrary to the estimators that have been considered in these papers we are also able to prove efficiency for our recursive estimators. This is not very surprising since our estimators turn out to be almost equal to the off-line maximum likelihood estimators, for which this property is well-known.

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**PROBABILISTIC AND WORST CASE ANALYSES OF  
CLASSICAL PROBLEMS OF COMBINATORIAL  
OPTIMIZATION IN EUCLIDEAN SPACE**

by

**J. Michael Steele**

*Princeton University  
Princeton, NJ*

**ABSTRACT**

The classical problems which are considered are the traveling salesman problem (TSP), minimal spanning tree (MST), minimal matching, greedy matching, and minimal triangulation. Each of these problems is studied for finite sets of points in  $\mathbb{R}^d$ , and the feature of principal interest is the asymptotic behavior of the value of the associated objective function.

# INFINITELY DIVISIBLE SEQUENCES AND RENEWAL SEQUENCES

by

B. G. Hansen

and

F. W. Steutel

*University of Technology  
Eindhoven, The Netherlands*

## ABSTRACT

Let  $(p_n)$  and  $(r_n)$  be related by

$$(1) \quad (n+1)p_{n+1} = \sum_{k=0}^n p_k r_{n-k} \quad (n = 0, 1, \dots),$$

i.e., if  $r_k \geq 0$  for all  $k$  then  $(p_n)$  is inf div. We prove two theorems.

**Theorem 1.** Let  $(p_n)$  satisfy (1), let  $M(I)$  denote the set of moment sequences with support in  $I$ , and let  $M^*(I)$  be the subset of  $M(I)$  with measures bounded by Lebesgue measure. Then for  $I \in \{\mathbb{R}_+[0, 1]\}$

$$(p_{n+1}) \in M(\mathbb{R}) \text{ iff } \left(\frac{r_n}{n+1}\right) \in M^*(\mathbb{R}) : (p_n) \in M(I) \text{ iff } \left(\frac{r_n}{n+1}\right) \in M^*(I).$$

**Theorem 2.** Let  $(p_n)$  satisfy (1) with  $r_k \geq 0$  for all  $k$  and let  $r_{-1} = 1$ . Then

(a) If  $(r_k)_{k=1}^\infty$  is log-convex, then  $(p_n)_0^\infty$  is log-convex.

(b) If  $(r_k)_{k=1}^\infty$  is log-concave, then  $(p_n)_0^\infty$  is log-concave (strongly unimodal).

Theorem 1 is the analogue of a theorem of Horn [2] for renewal sequences, whereas

(a) and (b) of theorem 2 are analogues of results by De Bruijn and Erdős [1] for renewal sequences and of Yamazato [3] for inf div densities.

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# QUALITY CONTROL IN ADVANCED MANUFACTURING SYSTEMS

by

Charles S. Tapiero

*Case Western Reserve University  
Cleveland, OH*

## ABSTRACT

The purposes of this seminar are twofold: (a) provide a conceptual framework for quality control in job-shop queueing like manufacturing systems, (b) outline several solved cases, including the quality control of job-shop  $M/M/1$ ,  $M/G/1$  queues, as well as FMS Open Queueing Networks. Implications for the quality control-inspection plans of automated manufacturing processes, flexible manufacturing systems and robotized cells are drawn. To conclude, an approach for joint quality and quantity production is suggested.



# VON MISES STATISTICS FOR STRONGLY DEPENDENT RANDOM VARIABLES

by

Murad S. Taqqu

*Boston University  
Boston, MA*

## ABSTRACT

Let  $(X_i)_{i=1}^{\infty}$  be a stationary, mean-zero Gaussian process with covariances  $f(k) = EX_{k+1} X_1$  satisfying  $r(0) = 1$  and  $r(k) = k^{-D} L(k)$  where  $D$  is small. consider the two-parameter empirical process for  $G(X_i)$ ,

$$\left\{ F_N(x, t) = \frac{1}{N} \sum_{i=1}^{\lfloor Nt \rfloor} 1\{G(X_i) \leq x\}; -\infty < x < +\infty, 0 \leq t \leq 1 \right\}$$

where  $G$  is any measurable function. Non-central limit theorems are obtained for  $F_N(x, t)$  and they are used to derive the asymptotic behavior of suitably normalized Von-Mises statistics

$$\sum_{1 \leq i_1, \dots, i_h \leq \lfloor Nt \rfloor} h(G(X_{i_1}), G(X_{i_2}), \dots, G(X_{i_h}))$$

where  $h$  has bounded total variation. Similar results hold for the  $U$ -statistics, where the summation excludes hyperdiagonals.

# INSENSITIVITY WITHOUT INSTANTANEOUS ATTENTION

by

P. Taylor

*University of Adelaide  
Adelaide, South Australia*

## ABSTRACT

Many authors have shown that partial balance is a necessary and sufficient condition for insensitivity in a stochastic process under the assumption that the process has the property of instantaneous attention, that is that lifetimes once created must immediately be worked on at a positive speed. It is possible to turn a process that does not have the property of instantaneous attention into a process that does by adding suitable extra states. However this paper takes a different approach to processes without instantaneous attention and derives balance equations which are necessary and sufficient for insensitivity.

It turns out that in processes that do have the property of instantaneous attention these balance equations are equivalent to the normal partial balance equations, but they can still hold without instantaneous attention.

# CONDITIONAL AND PATTERN - ORIENTED ROBUSTNESS

by

H. N. Teodorescu

*Institut Politehnic, Iasi  
Romania*

## ABSTRACT

The communication deals with the problem of robustness, as defined in filtering applications, and introduces two definitions of robustness related to applications oriented to pattern recognition.

In the first definition, it is assumed an application is defined from the set of 'signals' to a set on 'patterns', and a distance is given on the last set. Then, the robust filtering realizes the min max of the distance between the obtained and the true pattern.

In the second definition, the conditional probability the signal corresponds to a pattern is used.

Some basical properties of the above defined robustness are also given.

# INSPECTION POLICIES FOR DETERIORATING UNITS WITH SYMPTOMATIC EMISSIONS

by

L. C. Thomas

*University of Edinburgh  
Edinburgh, UK*

D. P. Gaver

and

P. A. Jacobs

*Naval Postgraduate School  
Monterey, CA*

## ABSTRACT

This paper considers the problem of when to inspect a unit which can deteriorate and fail to operate satisfactorily. Most previous work in this area assumes either that any change in the state of the unit is immediately noticed (continuous inspection) or that on inspection the state of the unit is known exactly. The models in this paper consider the case where on inspection the state of the unit is not known but the values of related phenomena, like ill pressure or vibration are obtained. From the changes in these readings the decision has to be made whether the unit is in a satisfactory state or whether it is necessary to repair or replace it.

One class of the models looks at the case of stand-by units where the unit is only in operation in emergencies and the inspections are trying to ascertain whether the unit has already "failed" and so will not perform adequately in an emergency. A second set of models look at active units where the inspections are trying to determine if the units have degraded to a point where their failure is imminent and so replacement or repair is necessary.

Markov decision process theory is used to determine which types of policies are optimal and renewal theory used to determine the characteristics of such policies.

**THE INITIAL TRANSIENCE PROBLEM:  
SOLUTION IN THE BOUNDED CYCLE LENGTH CASE**

by

**Hermann Thorisson**

*Chalmers University of Technology  
Goteborg, Sweden*

**ABSTRACT**

Consider the simulation problem of generating a stationary version of a regenerative process when it is known how to generate the i.i.d. cycles.

We present a solution of this problem in the bounded cycle length case. The solution is based on the so called acceptance/rejection method.

# OPTIMAL SWITCHING BETWEEN A PAIR OF BROWNIAN MOTIONS

by

R. J. Vanderbei

*AT&T Bell Laboratories  
Murray Hill, NJ*

## ABSTRACT

Consider a pair of Brownian motions  $X_t$  and  $Y_t$  on the interval  $[0, 1]$  with absorption at the endpoints. The joint state space is the square  $E = [0, 1] \times [0, 1]$ . The time evolution of the two processes can be controlled separately: i.e., we can let the  $X_t$  process run and freeze the  $Y_t$  process to obtain horizontal Brownian motion, or we can let the  $Y_t$  process run and freeze  $X_t$ , giving us vertical Brownian motion. We assume that there is a pay-off function  $f(x, y)$  that is zero in the interior of  $E$  and non-negative on the boundary of  $E$ . The objective is to find the optimal strategy for controlling the time evolution and a corresponding stopping time so as to maximize the expected pay-off obtained at the time of stopping. The optimal strategy is determined by a partition of the state space into three sets: horizontal control, vertical control, and stop. We will give a rather explicit characterization of these sets.

# ALGEBRAIC DUALITY OF MARKOV PROCESSES

by

Wim Vervaat

*Catholic University at Nijmegen*

## ABSTRACT

Let  $\eta = (\eta_t)_{t \geq 0}$  and  $\zeta = (\zeta_t)_{t \geq 0}$  be Markov processes with state spaces  $Y$  and  $Z$ . Following Liggett (1985) we say that  $\eta$  and  $\zeta$  are dual with respect to a function  $H : Y \times Z \rightarrow \mathbf{R}$  or  $\mathbf{C}$  if

$$\mathbf{E}_{\eta_0=y} H(\eta_t, z) = \mathbf{E}_{\zeta_0=z} H(y, \zeta_t)$$

for  $t \geq 0$  and  $(y, z) \in Y \times Z$ . It turns out that in all cases studied so far  $Y$  and  $Z$  are commutative semigroups and  $H$  is a homomorphism from  $Y \times Z$  into  $\mathbf{R}$  or  $\mathbf{C}$  with multiplication:

$$H(y_1 + y_2, z) = H(y_1, z)H(y_2, z),$$

$$H(y, z_1 + z_2) = H(y, z_1)H(y, z_2);$$

$H$  is said to be a duality between the semigroups  $Y$  and  $Z$ .

Examples of semigroups in duality will be discussed. One simple is:  $Y = [0, 1]$  with  $+$  = supremum;  $Z = [0, 1]$  with  $+$  = infimum;  $H(y, z) = 1$  if  $y \leq z$ , 0 else.

A general class of Markov processes in duality will be presented in this context. Both  $\eta$  and  $\zeta$  are random walks with state-dependent clock, stopping at the zeros of the semigroups  $Y$  and  $Z$ , so that these zeros are absorbing states. The clock speed of one random walk is the logarithm of the characteristic function of the steps of the other random walk. The special case of the groups  $Y = Z^d$ ,  $Z = (\mathbf{R}/\mathbf{Z})^d$  with  $+$  = addition and  $H(y, z) = e^{2\pi i y \cdot z}$  has been considered by Holley & Stroock (1979).

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# QUICK SIMULATIONS OF NETWORKS OF GI/GI/1 QUEUES

by

Jean Walrand

*University of California  
Berkeley, CA*

## ABSTRACT

A heuristic to speed up simulations of large backlogs in networks of GI/GI/1 queues is proposed. We use an importance sampling technique where the change of measure is based on large deviation estimates.

The change of measure is performed so that the modified system is again a network of GI/GI/1 queues. The service times and interarrival times (or the independent renewal arrival processes) are modified by an exponential change of measure. The routing probabilities are also modified. The calculations required for finding the new measures depend only on the number of queues, not on the backlog level to be investigated. Also, the necessary likelihood ratio calculations are recursive and inexpensive.

We present simulation results (in the case of Jackson networks, for simplicity) that demonstrate significant speed-ups.



# LARGE DEVIATIONS OF JUMP MARKOV PROCESSES WITH FLAT BOUNDARIES

by

Alan Weiss

*AT&T Bells Laboratories  
Murray Hill, NJ*

## ABSTRACT

Large deviations come up naturally in the study of certain telecommunication and queueing systems. These systems usually have boundaries, such as finite queue sizes or transmission capacities, which make it difficult to determine rate functions. In this paper, we show how to calculate the rate function when the boundaries may be represented as hyperplanes, the usual situation in practice. (We must assume that the sample paths avoid the corners and edges of the system, which remain subjects for future work.) We illustrate our results with examples from queueing and control.

In the special case where all jumps are either parallel to the boundary or normal to it, the theorem is a simple consequence of Donsker and Varadhan's work on the large deviations of local time. In general, the lower estimate (for open sets) is easy to prove since the optimal change of measure is not hard to guess. Surprisingly, it is the upper estimate that requires a bit of work, mainly in obtaining explicit expressions for some associated exponential transforms (moment generating functions). A minimax theorem then shows the equivalence of the upper and lower bounds.

# CENTRAL-LIMIT-THEOREM VERSIONS OF $L = \lambda W$

by

P. W. Glynn

*University of Wisconsin  
Madison, WI*

and

W. Whitt

*AT&T Bell Laboratories  
Murray Hill, NJ*

## ABSTRACT

Underlying the fundamental queueing formula  $L = \lambda W$  (Little's Law) is a relation between cumulative processes in continuous time (the integral of the queue length process) and in discrete time (the sum of the waiting times of successive customers). In addition to the familiar relation between the w.p.1 limits of the averages, there are corresponding relations among the central-limit-theorems (CLTs) [2,4]. Roughly speaking, the sequence of customer waiting times and interarrival times obey a joint CLT if and only if the continuous-time queue length and arrival counting process obey a joint CLT, in which case all four processes obey a joint CLT and the marginal limits are simply related. Similar results hold for extensions of  $L = \lambda W$  such as  $H = \lambda G$  [5].

The CLTs can be applied to compare the asymptotic efficiency of different estimators of queueing parameters [1,3]. More generally, such indirect estimation methods can be viewed as nonlinear control-variable estimators, which are asymptotically equivalent to linear control-variable estimators [3].

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# SUBORDINATION OF STATIONARY PROCESSES

by

Eric Willekens\*

and

Jozef L. Teugels

## ABSTRACT

Let  $X = \{X(t), t \in T \subset \mathbb{R}\}$  be a stationary process and suppose that  $N = \{N(t), t \geq 0\}$  is an infinitely divisible process, independent of  $X$ . Then the process  $\hat{X} := \{\hat{X}(t) = X(N(t)), t \geq 0\}$  is called subordinated to  $X$  (or derived from  $X$ ) with subordinator  $N$ . We show that  $\hat{X}$  is again a stationary process and we relate the spectral properties of  $X$  and  $\hat{X}$  by comparing their spectral measures. We obtain among others that if  $X$  is stochastically continuous

$$\hat{f}(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\operatorname{Re} \varphi(u)}{(\operatorname{Re} \varphi(u))^2 + (x + \operatorname{Im} \varphi(u))^2} f(u) du, \quad -\infty < x < \infty.$$

Here  $f$  and  $\hat{f}$  are the resp. spectra of  $X$  and  $\hat{X}$  and  $\varphi(u) = -\log E(e^{iuN(1)})$ . We also discuss the possibility of a derived stationary process to model time series in random time domains and give several examples.

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\* Research assistant of the Belgian National Fund for Scientific Research.

# BROWNIAN MODELS OF OPEN QUEUEING NETWORKS: PRODUCT-FORM STATIONARY DISTRIBUTIONS\*

by

R. J. Williams

*University of California  
San Diego, CA*

## ABSTRACT

We consider a class of multidimensional diffusion processes that arise as heavy traffic approximations for open queueing networks. A necessary and sufficient condition is derived for such a process to have a stationary distribution with a separable (product form) density. When that condition is satisfied, the stationary distribution is exponential and all standard performance measures can be written out in explicit formulas.

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\* Based on joint work with J. M. Harrison.

# A PATHWISE APPROACH TO STOCHASTIC INTEGRATION

by

Walter Willinger

*Bell Communications Research*

## ABSTRACT

We develop a pathwise construction of stochastic integrals relative to continuous martingales. The key to the construction is an almost-sure approximation technique which associates to a given continuous martingale and its minimal filtration a sequence of finitely generated filtrations ("skeleton-filtrations") and a sequence of simple stochastic processes ("skeletons"). A pathwise stochastic integral can then be defined along such a "skeleton-approximation." Almost-sure convergence follows from a certain completeness property along the skeleton-approximation. The limit is the pathwise stochastic integral and it agrees with the integral obtained through the usual Ito-approach.

We apply the skeleton-technique and pathwise stochastic integration to the theory of security markets with continuous trading. A convergence theory for continuous market models is obtained which enables us to view an idealized economy (i.e. a continuous market model) as an almost-sure limit of "real-life" economies (i.e. finite market models).

# LIMIT THEOREMS FOR DOWNCROSSINGS OF A CLASS OF BIRTH AND DEATH PROCESSES

by

**Keigo Yamada**

*University of Tsukuba  
Ibaraki, Japan*

## ABSTRACT

For the number of downcrossings of Brownian motions, there is a strong law type of limit theorem due to Levy. A central limit theorem version of Levy's results was obtained by Y. Kasahara.

In this paper we obtain similar results for a class of birth and death processes. The necessity for results of this type was occasioned by investigation of some problems of queueing theory.

# NOTES ON THE CLASSICAL DYNKIN STOPPING PROBLEM

by

M. Yasuda

*Chiba University  
Chiba, Japan*

## ABSTRACT

E. B. Dynkin (1969) and E. B. Frid (1969) proposed a game version of the stopping problem for Markov processes, in which strategies are subject to the condition of the prescribed disjoint subsets in the state space. Under this condition the simultaneous stopping does not occur and so, switching the move of the game makes the problem simple. We shall show that some problems of the classical Dynkin stopping problem could be reduced to the standard optimal stopping problems for the same Markov process. As a related topic, the singular stochastic control (impulse control) is considered.

*Keywords:* Optimal stopping, Dynkin game, stochastic control.

# STOCHASTIC SEQUENTIAL ASSIGNMENT BASED ON DISCRETE MATCH-LEVELS

by

Israel David

*Tel Aviv University  
Tel Aviv, Israel*

and

Uri Yechiali

*New York University  
New York, NY*

## ABSTRACT

$M$  "offers" (e.g. kidneys for transplants) arrive in a random stream and are to be sequentially assigned to  $N$  waiting candidates. Each candidate, as well as each arrival, is characterized by a random attribute drawn from a discrete-valued probability distribution function. An assignment of an offer to a candidate yields a reward  $r(o)$  if they match, and a reward  $r(1) \leq r(o)$  if not. We derive optimal sequential assignment policies which maximize the expected total reward for various cases where  $M \neq N$  and various decay assumptions on the underlying life time distribution of the process. We give intuitive explanations for these optimal strategies and indicate applications.



# PRINCIPAL VALUES OF BROWNIAN LOCAL TIMES

by

M. Yor

and

Ph. Biane

*Université Pierre et Marie Curie  
Paris, France*

## ABSTRACT

We study the Cauchy principal value of brownian local times. We give its law, when taken at a fixed time as well as random times; the calculations use Ito's excursion theory and the results are closely linked with P. Levy's stochastic area formula. We are then led to consider identities which relate the laws of the brownian bridge, excursion and meander, their local times and their maxima.

Results contained in [1] will be summarized while new results will be presented in more detail.

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# UNIVERSAL LIMIT THEOREMS FOR THE FUNCTION INDEXED EMPIRICAL PROCESS

by

J. E. Yukich

*Lehigh University  
Bethlehem, PA*

## ABSTRACT

Let  $\mathcal{F}$  be a class of functions on a measurable space  $(X, \mathcal{A})$ . Using a new combinatorial criteria, we provide probabilistic and combinatorial characterizations of those  $\mathcal{F}$  for which the function indexed empirical process  $(P_n - P)(f), f \in \mathcal{F}$ , satisfies uniform weak and strong laws of large numbers uniformly over all probability measures  $P$  on  $X$ . It is shown that the criteria, called Rademacher stability, effectively measures the oscillations of arbitrary function classes and plays a unifying role in the study of general universal limit theorems, including central limit theorems. The criteria extends the classic  $VC$  criteria to function classes and is closely related to the Talagrand-Fremlin notion of a stable function class; among other things we use it to provide a combinatorial characterization of those  $\mathcal{F}$  satisfying the bounded central limit theorem uniformly over all  $P$ .

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